Lösungen tegut T 16

<u>1.Aufgabe</u>

a) $f(x) = \frac{1}{2}x^3 - x^2 - \frac{5}{2}x$

D=R $\quad x \to -\infty; f(x) \to -\infty$

$x \to +\infty; f(x) \to +\infty$

Keine Symmetrie, da gerade und ungerade Exponenten vorhanden sind.

$S_y(0|0)$

$f(x) = 0$

$0 = \frac{1}{2}x^3 - x^2 - \frac{5}{2}x$ normalisieren, x ausklammern, pq-Formel

$x_1 = 0$; $x_2 \approx 3{,}45$; $x_3 \approx -1{,}45$

$S_{x1}(0|0)$ $\quad S_{x2}(3{,}45|0)$ $\quad S_{x3}(-1{,}45|0)$

$f'(x) = \frac{3}{2}x^2 - 2x - \frac{5}{2}$

$f''(x) = 3x - 2$

$f'''(x) = 3$

$f'(x) = 0$

$0 = \frac{3}{2}x^2 - 2x - \frac{5}{2}$ normalisieren, pq-Formel

$x_1 \approx 2{,}12$; $x_2 \approx -0{,}79$

$f'(x) = 0 \wedge f''(x) \neq 0$

$f''(2{,}12) = 4{,}36 > 0 \Rightarrow T$

$f''(-0{,}79) = -4{,}37 < 0 \Rightarrow H$

$f(2{,}12) \approx -5{,}03$ $\quad f(-0{,}79) \approx 1{,}10$

$T(2{,}12|-5{,}03)$ $\quad H(-0{,}79|1{,}10)$

$f''(x) = 0$

$0 = 3x - 2$

$x = \frac{2}{3}$

$f''(x) = 0 \wedge f'''(x) \neq 0$

$f'''\left(\frac{2}{3}\right) = 3 > 0 \Rightarrow R - L - K$

$f\left(\frac{2}{3}\right) \approx -1{,}96$

$W_{R-L}(0{,}67|-1{,}96)$

b) $x_1 = -1$ und $t(x) = m \cdot x + b$

$f(-1) = 1$ y-Wert

$f'(-1) = 1$ Steigung m $f'(x) = m$

$1 = 1 \cdot (-1) + b$

$b = 2$

$t(x_1) = x + 2$

c) $f(x) = t(x_1)$

$\frac{1}{2}x^3 - x^2 - \frac{5}{2}x = x + 2$

$0{,}5x^3 - x^2 - 3{,}5x - 2 = 0$

TR: $x_{1/2} = -1$ Tangentenstelle und $x_3 = 4$

$t(4) = 6$

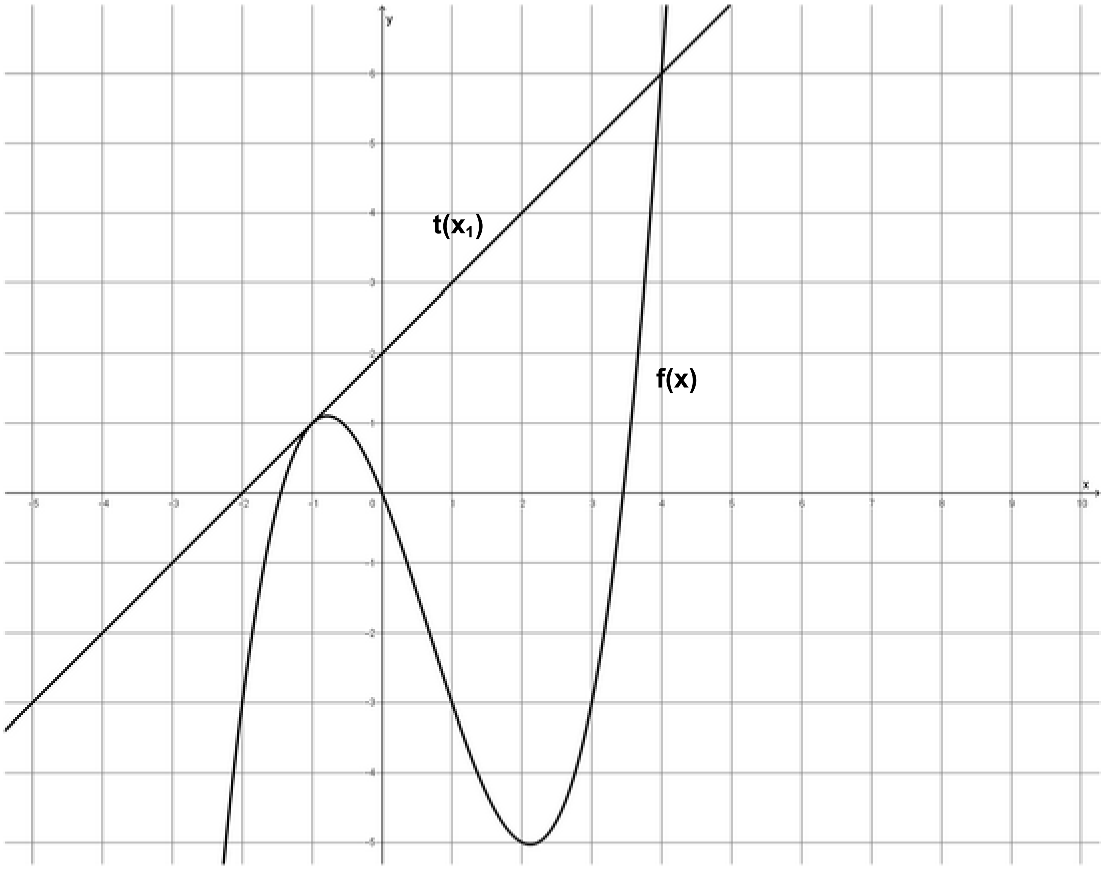
$S_3\left(4|6\right)$ weiterer Schnittpunkt

d) $f(-2) = -3$

$f(1) = -3$

$f(3) = -3$

e)



f) $A_1 = \int_{-1,45}^{0} \left(\frac{1}{2}x^3 - x^2 - \frac{5}{2}x\right)dx$

$A_1 = \left[\frac{1}{8}x^4 - \frac{1}{3}x^3 - \frac{5}{4}x^2\right]_{-1,45}^{0}$

$A_1 = [0] - [-1{,}06]$

$A_1 = 1{,}06\text{FE}$

$A_2 = \left|\int_{0}^{3,45} \left(\frac{1}{2}x^3 - x^2 - \frac{5}{2}x\right)dx\right|$

$A_2 = \left|\left[\frac{1}{8}x^4 - \frac{1}{3}x^3 - \frac{5}{4}x^2\right]_{0}^{3,45}\right|$

$A_2 = \left|[-10{,}86] - [0]\right|$

$A_2 = |-10{,}86| = 10{,}86\text{FE}$

$A = A_1 + A_2 = 1{,}06 + 10{,}86 = 11{,}92\text{FE}$

g) $A = \left| \int_{-1}^{4} \left(\frac{1}{2}x^3 - x^2 - \frac{7}{2}x - 2 \right) dx \right|$ TR: $A = \left| -\frac{625}{24} \right| = \frac{625}{24}$ FE

h) $[S_x t(x_1); S_1]$ => $t(x_1) = 0$ => $x = -2$ => $[-2; -1]$

$A_1 = \int_{-2}^{-1,45} (x+2)\,dx$

$A_1 = \left[\frac{1}{2}x^2 + 2x \right]_{-2}^{-1,45}$

$A_1 = \left[-\frac{1479}{800} \right] - [-2]$

$A_1 = \frac{121}{800}$ FE

$A_2 = \left| \int_{-1,45}^{-1} \left(\frac{1}{2}x^3 - x^2 - \frac{7}{2}x - 2 \right) dx \right|$

$A_2 = \left| \left[\frac{1}{8}x^4 - \frac{1}{3}x^3 - \frac{7}{4}x^2 - 2x \right]_{-1}^{-1,45} \right|$

$A_2 = \left| [0,79] - \left[\frac{17}{24} \right] \right|$

$A_2 = \left| -\frac{49}{600} \right| = \frac{49}{600}$ FE

$$A = A_1 + A_2 = \frac{121}{800} + \frac{49}{600} = \frac{26}{75} \text{ FE}$$

i) $A_2 = \int_{-1}^{0} \left(\frac{1}{2}x^3 - x^2 - \frac{7}{2}x - 2 \right) dx$

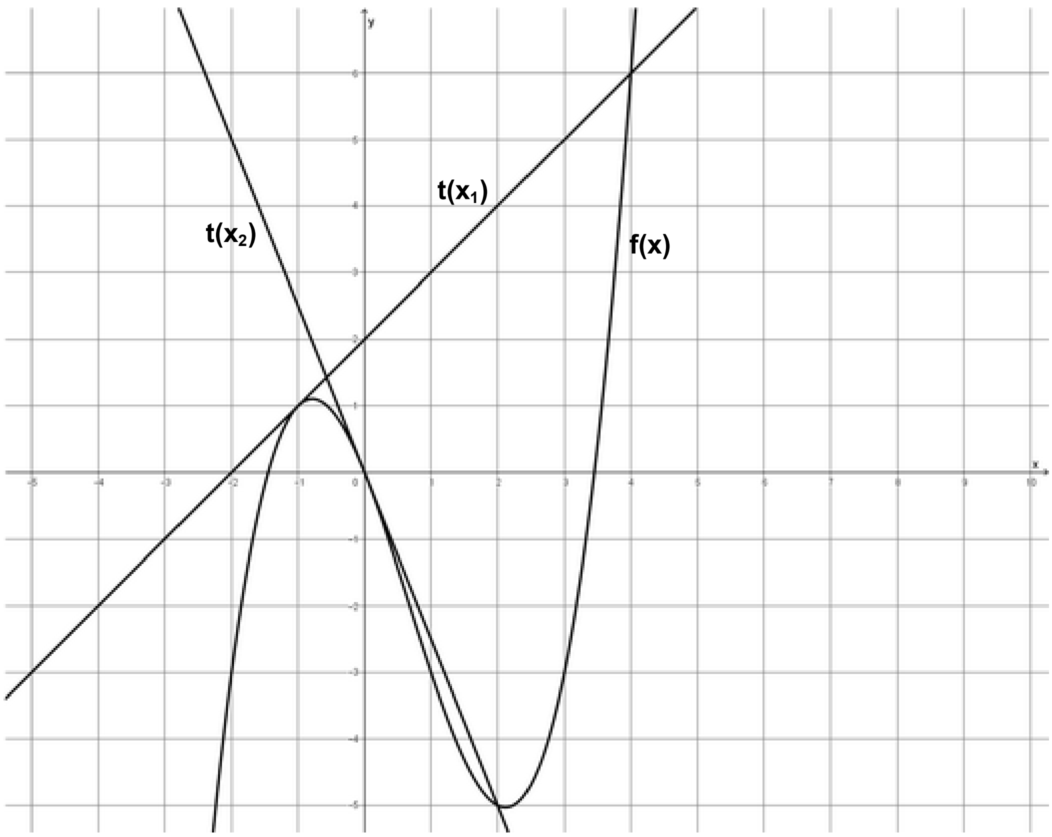
$A_3 = \frac{17}{24}$ FE

j) Tangente im Ursprung $(0|0)$ => $b = 0$

$f'(x) = m$ mit $f'(0) = -2,5$

$t(x_2) = -2,5x$

k)



l) $f(x) = t(x_2)$

$$\frac{1}{2}x^3 - x^2 - \frac{5}{2}x = -\frac{5}{2}x$$

$$0{,}5x^3 - x^2 = 0$$

$$x^2(0{,}5x - 1) = 0$$

$x_{1/2} = 0$ Tangentenstelle und $x_3 = 2$

$$A = \int_0^2 \left(t(x_2) - f(x)\right) dx$$

$$A = \int_0^2 \left(-0{,}5x^3 + x^2\right) dx$$

$$A = \left[-\frac{1}{8}x^4 + \frac{1}{3}x^3\right]_0^2$$

$$A = \left[\frac{2}{3}\right] - [0]$$

$$A = \frac{2}{3}\text{FE}$$

m)