Lösungen U 12

Aufgabe 1

a) $f(x) = \frac{1}{20}x^4 - \frac{3}{10}x^3$

mit Kommazahlen: $f(x) = 0{,}05x^4 - 0{,}3x^3$

1 $f'(x) = \frac{1}{5}x^3 - \frac{9}{10}x^2$ $\qquad f'(x) = 0{,}2x^3 - 0{,}9x^2$

$f''(x) = \frac{3}{5}x^2 - \frac{9}{5}x$ $\qquad f''(x) = 0{,}6x^2 - 1{,}8x$

$f'''(x) = \frac{6}{5}x - \frac{9}{5}$ $\qquad f'''(x) = 1{,}2x - 1{,}8$

2 $x \to -\infty;\ f(x) = +\infty$

$x \to +\infty;\ f(x) = +\infty$

3 KS

4 $f(x) = 0$

$0 = \frac{1}{20}x^4 - \frac{3}{10}x^3 \quad |:\frac{1}{20}$

$0 = x^4 - 6x^3$

$0 = x^3(x-6)$

$x_{1/2/3} = 0 \qquad x_4 = 6$

$S_y(0|0)$

$S_{x_{1/2/3}}(0|0)$

$S_{x_4}(6|0)$

5 $f'(x) = 0$ und $f''(x) \neq 0$

$0 = \frac{1}{5}x^3 - \frac{9}{10}x^2 \quad |:\frac{1}{5}$

$0 = x^3 - 4{,}5x^2$

$0 = x^2(x-4{,}5)$

$x_{1/2} = 0 \qquad x_3 = 4{,}5$

$f''(0) = 0 = 0 \Rightarrow$ Sattelpunkt

$f''(4{,}5) = 4{,}1 > 0 \Rightarrow$ Min

$f(0) = 0 \quad SP(0|0)$

$f(4{,}5) = -6{,}8 \quad TP(4{,}5|-6{,}8)$

6 $f''(x) = 0$ und $f'''(x) \neq 0$

$0 = \frac{3}{5}x^2 - \frac{9}{5}x \quad |:\frac{3}{5}$

$0 = x^2 - 3x$

$0 = x(x-3)$

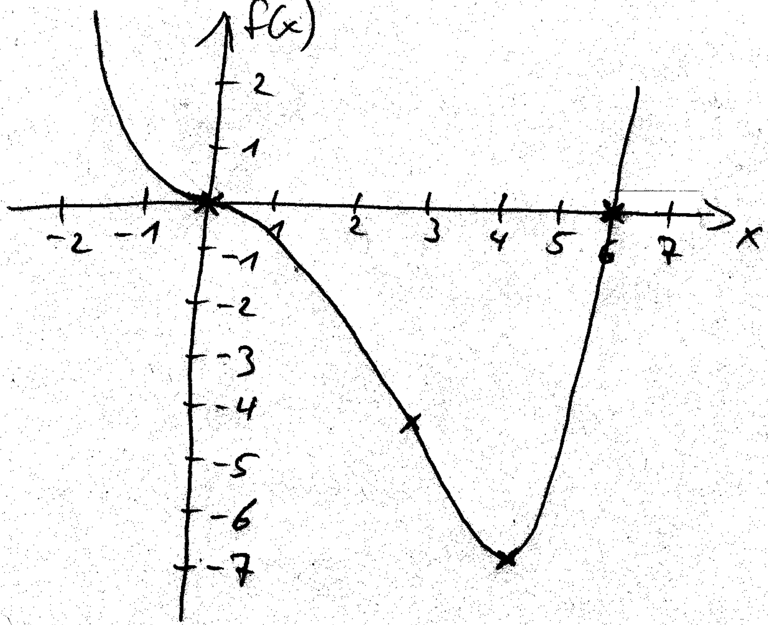
$x_1 = 0 \qquad f'''(0) = -1{,}8 < 0 \Rightarrow$ L-R-k

$x_2 = 3 \qquad f'''(3) = 1{,}8 > 0 \Rightarrow$ R-L-k

$f(0)=0 \quad W_{L-R}(0|0)$ 2

$f(3)=-4{,}1 \quad W_{R-L}(3|-4{,}1)$ U12

7 Skizze



b) $m=2{,}5 \quad f'(x)=m$

$2{,}5=0{,}2x^3-0{,}9x^2 \quad |-2{,}5$

$0=0{,}2x^3-0{,}9x^2-2{,}5 \quad |:0{,}2$

$0=x^3-4{,}5x^2-12{,}5 \qquad \underline{\underline{x_1=5}}$

$(x^3-4{,}5x^2+0x-12{,}5):(x-5)=x^2+0{,}5x+2{,}5$

$\underline{-(x^3-5x^2)}$

$0{,}5x^2+0x$

$\underline{-(0{,}5x^2-2{,}5x)}$

$2{,}5x-12{,}5$

$\underline{-(2{,}5x-12{,}5)}$

0

$x^2+0{,}5x+2{,}5=0$

$x_{2/3}=-0{,}25\pm\sqrt{0{,}25^2-2{,}5}$

$x=5 \quad f(5)=-6{,}25 \; y$

$m=2{,}5$

$t(x)=m\cdot x+b$

$-6{,}25=2{,}5\cdot 5+b \quad |-12{,}5$

$-18{,}75=b$

$\underline{\underline{t_1(x)=2{,}5x-18{,}75}}$

c) $x = -2 \quad f(-2) = 3{,}2 \;\; y$

$f'(-2) = -5{,}2 \;\; m$

$t(x) = m \cdot x + b$

$3{,}2 = -5{,}2 \cdot (-2) + b \quad | -10{,}4$

$-7{,}2 = b$

$$t_2(x) = -5{,}2x - 7{,}2$$

d) $t_1(x) = t_2(x)$

$2{,}5x - 18{,}75 = -5{,}2x - 7{,}2 \quad | +5{,}2x + 18{,}75$

$7{,}7x = 11{,}55 \quad | : 7{,}7$

$x = 1{,}5$

$t_1(1{,}5) = -15 \qquad \underline{S(1{,}5 \mid -15)}$

e) $SP(\underset{x_1}{0} \mid \underset{y_1}{0}) \qquad S(\underset{x_2}{1{,}5} \mid \underset{y_2}{-15})$

$d = \sqrt{(1{,}5-0)^2 + (-15-0)^2}$

$d = 15{,}1$ LE

Aufgabe 2

a) $f(x) = -x^3 + 2x^2 + 13x + 10$

$f(x) = 0$

$0 = -x^3 + 2x^2 + 13x + 10 \quad | : (-1)$

$0 = x^3 - 2x^2 - 13x - 10 \qquad \underline{x_1 = -1}$

$(x^3 - 2x^2 - 13x - 10) : (x+1) = x^2 - 3x - 10$

$-(x^3 + 1x^2)$

$-3x^2 - 13x$

$-(-3x^2 - 3x)$

$-10x - 10$

$-(-10x - 10)$

0

$x^2 - 3x - 10 = 0$

$x_{2/3} = +1{,}5 \pm \sqrt{2{,}25 + 10}$

$\underline{x_2 = 5}$

$\underline{x_3 = -2}$

$$A_1 = \left| \int_{-2}^{-1} (-x^3 + 2x^2 + 13x + 10)\,dx \right|$$

$$= \left| \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + 6{,}5x^2 + 10x \right]_{-2}^{-1} \right|$$

$$= \left| [-4{,}4] - [-3\tfrac{1}{3}] \right| = |-1{,}1| = \underline{\underline{1{,}1\,FE}}$$

$$A_2 = \int_{-1}^{5} (-x^3 + 2x^2 + 13x + 10)\,dx$$

$$= \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + 6{,}5x^2 + 10x \right]_{-1}^{5}$$

$$= [139{,}6] - [-4{,}4] = \underline{\underline{144\,FE}}$$

$$A_{gesamt} = A_1 + A_2$$

$$= 1{,}1 + 144 = \underline{\underline{145{,}1\,FE}}$$

b)

$f(x)$: $a = 0$; $b = 5$

$g(x)$: $a = 0$; $b =$ Nullstelle $\Rightarrow g(x) = 0$

$0 = -2x + 4$

$2x = 4$

$x = 2 \Rightarrow b = 2$

$$A_1 = \int_{0}^{5} (-x^3 + 2x^2 + 13x + 10)\,dx$$

$$= \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + 6{,}5x^2 + 10x \right]_{0}^{5} = [139{,}6] - [0]$$

$$= \underline{\underline{139{,}6\,FE}}$$

$$A_2 = \int_{0}^{2} (-2x + 4)\,dx = \left[-x^2 + 4x \right]_{0}^{2} = [4] - [0]$$

$$= \underline{\underline{4\,FE}}$$

$$A_{gesucht} = A_1 - A_2$$

$$= 139{,}6 - 4 = \underline{\underline{135{,}6\,FE}}$$

Alternativ zum Integrieren kann man das Dreieck, das durch $g(x)$ entsteht auch mit $A = \frac{1}{2} \cdot g \cdot h$ berechnen.

$g = 2$ (Nullstelle)
$h = 4$ (S_y)
$\Rightarrow A = \frac{1}{2} \cdot 2 \cdot 4$
$\underline{\underline{A = 4\,FE}}$

(weitere Rechnung siehe oben)

<u>Aufgabe 3</u>

a) $E'(x) = -12x + 42$
$E(x) = -6x^2 + 42x$
$P(x) = -6x + 42 \rightarrow HP = 42\,GE$
$P(x) = 0$
$0 = -6x + 42 \quad | +6x$
$6x = 42 \quad | :6$
$\underline{\underline{SM \quad x = 7\,ME}} \Rightarrow D_{ök} = [0;7]$

b) $K'(x) = 3x^2 - 18x + 30$
$K''(x) = 6x - 18$
$K'''(x) = 6$

$K''(x) = 0$ und $K'''(x) \neq 0$
$0 = 6x - 18 \quad | +18$
$18 = 6x \quad | :6$
$\underline{x = 3\,ME}$
$K'''(3) = 6 > 0 \Rightarrow Min$

$K'(3) = 3\,GE$

$Gk_{min}\,(3|3)$

Bei 3 ME findet die geringste Kostensteigerung mit 3 GE statt.

6 U12

c) Hier kann man direkt mit den Ableitungen arbeiten.

$G'(x) = E'(x) - K'(x)$

$G'(x) = -12x + 42 - (3x^2 - 18x + 30)$

$G'(x) = -12x + 42 - 3x^2 + 18x - 30$

$\underline{G'(x) = -3x^2 + 6x + 12}$

$G''(x) = -6x + 6$

$G'(x) = 0$ und $G''(x) \neq 0$

$0 = -3x^2 + 6x + 12 \quad | :(-3)$

$0 = x^2 - 2x - 4$

$x_{1/2} = +1 \pm \sqrt{1+4}$

$\underline{x_1 = 3{,}2 \text{ ME} \quad x_{Gmax}}$ $\quad G''(3{,}2) = -13{,}2 < 0 \Rightarrow \text{Max}$

$[x_2 = -1{,}2]$

$P(3{,}2) = 22{,}8 \text{ GE} \qquad C(3{,}2 \mid 22{,}8)$

d)

$K'(x) = 3x^2 - 18x + 30$

$K(x) = x^3 - 9x^2 + 30x + K_{fix} \qquad (2 \mid 42)$

$42 = 2^3 - 9 \cdot 2^2 + 30 \cdot 2 + K_{fix}$

$42 = 32 + K_{fix} \quad | -32$

$10 = K_{fix}$

$\underline{\Rightarrow K(x) = x^3 - 9x^2 + 30x + 10}$

$G(x) = E(x) - K(x)$

$= -6x^2 + 42x - (x^3 - 9x^2 + 30x + 10)$

$= -6x^2 + 42x - x^3 + 9x^2 - 30x - 10$

$\underline{\underline{G(x) = -x^3 + 3x^2 + 12x - 10}}$