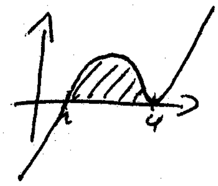
Lösungen N 12

Aufgabe 1

$f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x - 4$

Fläche mit x-Achse
$\Rightarrow$ Nullstellen



$f(x) = 0$

$0 = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x - 4 \quad | : \frac{1}{4}$

$0 = x^3 - 9x^2 + 24x - 16 \qquad x_1 = 1$

$$\begin{aligned}
&(x^3 - 9x^2 + 24x - 16) : (x-1) = x^2 - 8x + 16 \\
&\underline{-(x^3 - 1x^2)} \\
&\quad -8x^2 + 24x \\
&\quad \underline{-(-8x^2 + 8x)} \\
&\qquad 16x - 16 \\
&\qquad \underline{-(16x - 16)} \\
&\qquad\qquad 0
\end{aligned}$$

$x^2 - 8x + 16 = 0$

$x_{2/3} = +4 \pm \sqrt{16-16}$

$x_{2/3} = 4$

$$A = \int_1^4 \left(\frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x - 4\right)dx = \left[\frac{1}{16}x^4 - \frac{3}{4}x^3 + 3x^2 - 4x\right]_1^4$$

$$= [0] - \left[-\frac{27}{16}\right] = \frac{27}{16}\,FE \quad (= 1{,}7\,FE)$$

Aufgabe 2

$f(x) = x^4 - 20x^2 + 64$

Fläche zwischen den beiden Tiefpunkten
$\Rightarrow$ Extremwerte
$\Rightarrow$ Nullstellen

$f(x) = 0$

$0 = x^4 - 20x^2 + 64$

$x^2 = z$

$0 = z^2 - 20z + 64$

$z_{1/2} = +10 \pm \sqrt{100-64}$

$z_1 = 16 \qquad z = x^2 \qquad x^2 = 16 \,|\sqrt{\ } \qquad x_1 = 4 \quad x_2 = -4$

$z_2 = 4 \qquad\qquad\qquad x^2 = 4 \,|\sqrt{\ } \qquad x_3 = 2 \quad x_4 = -2$

$f'(x) = 0$ und $f''(x) \neq 0$

$f'(x) = 4x^3 - 40x$ $\qquad$ $0 = 4x^3 - 40x \quad |:4$

$f''(x) = 12x^2 - 40$ $\qquad$ $0 = x^3 - 10x$

$0 = x(x^2 - 10)$

$x_1 = 0$ $\qquad$ $x^2 - 10 = 0$

$x^2 = 10 \quad |\sqrt{\ }$

$x_2 = 3{,}2$

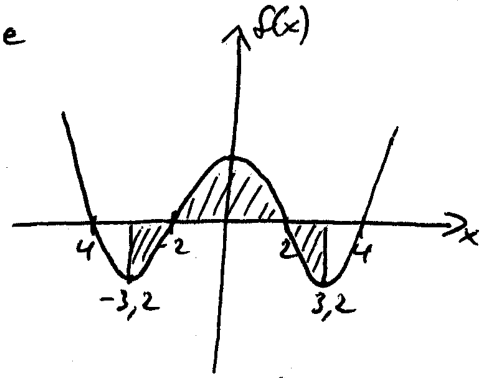
$x_3 = -3{,}2$

$f''(0) = -40 < 0 \Rightarrow HP$

$f''(3{,}2) = 82{,}88 > 0 \Rightarrow TP$

$f''(-3{,}2) = 82{,}88 > 0 \Rightarrow TP$

Skizze



Wegen AS kann man ab der y-Achse berechnen und jeweils mit 2 multiplizieren.

$$A_1 = 2 \cdot \int_0^2 (x^4 - 20x^2 + 64)\,dx = 2 \cdot \left[\tfrac{1}{5}x^5 - \tfrac{20}{3}x^3 + 64x\right]_0^2$$

$$= 2 \cdot \left(\left[81\tfrac{1}{15}\right] - [0]\right) = \underline{162\tfrac{2}{15}\ FE} \quad (162{,}1\ FE)$$

$$A_2 = 2 \cdot \left|\int_2^{3{,}2} (x^4 - 20x^2 + 64)\,dx\right| = 2 \cdot \left|\left[\tfrac{1}{5}x^5 - \tfrac{20}{3}x^3 + 64x\right]_2^{3{,}2}\right|$$

$$= 2 \cdot \left|\left([53{,}5] - \left[81\tfrac{1}{15}\right]\right)\right| = |-55{,}1| = \underline{55{,}1\ FE}$$

$$A_{gesamt} = A_1 + A_2$$

$$= 162{,}1 + 55{,}1 = \underline{\underline{217{,}2\ FE}}$$

Aufgabe 3

$f_1(x) = x^3 - 8x^2 + 16x$ $\qquad$ $f_1(x) = f_2(x)$

$f_2(x) = 0{,}4x^3 - 2{,}6x^2 + 4x$

$$x^3 - 8x^2 + 16x = 0{,}4x^3 - 2{,}6x + 4x \quad | -0{,}4x^3 + 2{,}6x - 4x$$

$$\underline{0{,}6x^3 - 5{,}4x^2 + 12x = 0} \quad | : 0{,}6$$

$$x^3 - 9x^2 + 20x = 0$$

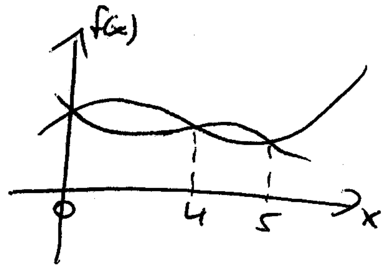
$$x(x^2 - 9x + 20) = 0$$

$$x_1 = 0 \qquad x^2 - 9x + 20 = 0$$

$$x_{2,3} = +4{,}5 \pm \sqrt{4{,}5^2 - 20}$$

$$x_2 = 5$$

$$x_3 = 4$$



$$A_1 = \int_0^4 (0{,}6x^3 - 5{,}4x^2 + 12x)\,dx = \left[0{,}15x^4 - 1{,}8x^3 + 6x^2\right]_0^4$$

$$= [19{,}2] - [0] = \underline{19{,}2 \text{ FE}}$$

$$A_2 = \left|\int_4^5 (0{,}6x^3 - 5{,}4x^2 + 12x)\,dx\right| = \left|\left[0{,}15x^4 - 1{,}8x^3 + 6x^2\right]_4^5\right|$$

$$= \left|[18{,}75] - [19{,}2]\right| = |-0{,}45| = \underline{0{,}45 \text{ FE}}$$

$$A_{gesamt} = A_1 + A_2$$

$$= 19{,}2 + 0{,}45 = \underline{\underline{19{,}65 \text{ FE}}}$$

Aufgab 4

$$f(x) = 4x^3 - 6x \qquad [2; b]$$

$$A = \int_2^b (4x^3 - 6x)\,dx = \left[x^4 - 3x^2\right]_2^b$$

$$= [b^4 - 3b^2] - [4] \qquad A = 50 \text{ FE}$$

$$50 = b^4 - 3b^2 - 4 \quad | -50$$

$$0 = b^4 - 3b^2 - 54 \qquad b^2 = z$$

$0 = z^2 - 3z - 54$

$z_{1/2} = +1{,}5 \pm \sqrt{2{,}25 + 54}$

$z_1 = 9 \qquad z = b^2 \qquad b^2 = 9 \mid \sqrt{\ } \qquad \underline{\underline{b_1 = 3}} \quad [b_2 = -3]$

$z_2 = -6 \qquad\qquad\qquad b^2 = -6 \mid \sqrt{\ } \quad /$

Aufgabe 5

$f(x) = -x^2 + 2x + 3$ Fläche mit x- und y-Achse

$\Rightarrow$ Nst.

$f(x) = 0$

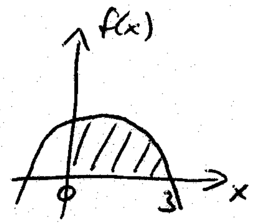
$0 = -x^2 + 2x + 3$

$0 = x^2 - 2x - 3$

$x_{1/2} = +1 \pm \sqrt{1+3}$

$x_1 = 3$

$[x_2 = -1]$



$$A = \int_0^3 (-x^2 + 2x + 3)\,dx = \left[-\tfrac{1}{3}x^3 + x^2 + 3x\right]_0^3$$

$$= [9] - [0] = \underline{\underline{9\,FE}}$$

Aufgabe 6

a) $t(x) = x + 21{,}5 \quad [0; 1]$

$$A = \int_0^1 (x + 21{,}5)\,dx = \left[\tfrac{1}{2}x^2 + 21{,}5x\right]_0^1 = [22] - [0]$$

$$\underline{\underline{= 22\,FE}}$$

b) $f(x) = -0{,}5x^2 + 11x - 28{,}5$ Fläche mit x-Achse

$f(x) = 0$

$0 = -0{,}5x^2 + 11x - 28{,}5 \qquad x_1 = 19$

$0 = x^2 - 22x + 57 \qquad x_2 = 3$

$x_{1/2} = 11 \pm \sqrt{121 - 57}$

$$A = \int_3^{19} (-0{,}5x^2 + 11x - 28{,}5)\,dx = \left[-\tfrac{1}{6}x^3 + 5{,}5x^2 - 28{,}5x\right]_3^{19}$$

$$= [300{,}8] - [-40{,}5] = \underline{\underline{341{,}3\ FE}}$$

c) Schnittpunkt von $t(x)$ und $f(x)$ bestimmen

$$t(x) = f(x)$$

$$x + 21{,}5 = -0{,}5x^2 + 11x - 28{,}5 \quad | -x - 21{,}5$$

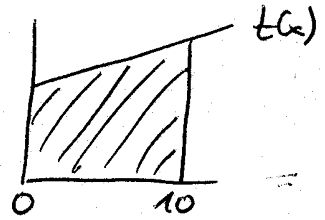
$$0 = -0{,}5x^2 + 10x - 50 \quad | : (-0{,}5)$$

$$0 = x^2 - 20x + 100$$

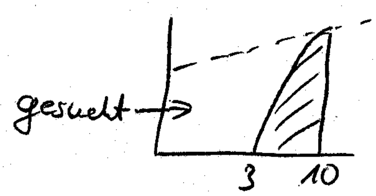
$$x_{1/2} = 10 \pm \sqrt{100 - 100}$$

$$x_{1/2} = 10$$

Fläche 1



Fläche 2



$$A_{gesucht} = A_1 - A_2$$

$$A_1 = \int_0^{10} (x + 21{,}5)\,dx = \left[\tfrac{1}{2}x^2 + 21{,}5x\right]_0^{10} = [265] - [0]$$

$$= \underline{265\ FE}$$

$$A_2 = \int_3^{10} (-0{,}5x^2 + 11x - 28{,}5)\,dx = \left[-\tfrac{1}{6}x^3 + 5{,}5x^2 - 28{,}5x\right]_3^{10}$$

$$\left[98\tfrac{1}{3}\right] - [-40{,}5] = \underline{138{,}8\ FE}$$

$$A_{gesucht} = 265 - 138{,}8$$

$$= \underline{\underline{126{,}2\ FE}}$$