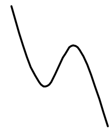
Lösungen D 14

<u>1. Aufgabe</u>

$f_1(x) = -0{,}5x^3 - 2x^2 - 0{,}5x + 3$ 1. $D = \mathbb{R}$ 2. $x \to -\infty;\ f(x) \to +\infty$, $x \to +\infty;\ f(x) \to -\infty$  3. KS

4. $S_y(0|3)$ $f(x) = 0$ $0 = -0{,}5x^3 - 2x^2 - 0{,}5x + 3 \quad |:(-0{,}5)$

$0 = x^3 + 4x^2 + x - 6$ Polynomdivision mit $x_1 = 1$

$$(x^3 + 4x^2 + x - 6) : (x - 1) = x^2 + 5x + 6$$

$$\begin{aligned} &-(x^3 - 1x^2) \\ &\quad 5x^2 + x \\ &\quad -(5x^2 - 5x) \\ &\qquad 6x - 6 \\ &\qquad -(6x - 6) \\ &\qquad\quad 0 \end{aligned}$$

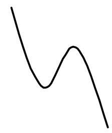
$x^2 + 5x + 6 = 0$ p-q-Formel

$x_{2/3} = -2{,}5 \pm \sqrt{2{,}5^2 - 6}$

$x_2 = -2$

$x_3 = -3$

$S_{x1}(1|0)$ $S_{x2}(-2|0)$ $S_{x3}(-3|0)$

$f_2(x) = -0{,}25x^3 - 2x^2 + 0{,}25x + 2$ 1. $D = \mathbb{R}$ 2. $x \to -\infty;\ f(x) \to +\infty$, $x \to +\infty;\ f(x) \to -\infty$  3. KS

4. $S_y(0|2)$ $f(x) = 0$ $0 = -0{,}25x^3 - 2x^2 + 0{,}25x + 2 \quad |:(-0{,}25)$

$0 = x^3 + 8x^2 - x - 8$ Polynomdivision mit $x_1 = 1$

$$(x^3 + 8x^2 - x - 8) : (x - 1) = x^2 + 9x + 8$$

$$\begin{aligned} &-(x^3 - 1x^2) \\ &\quad 9x^2 - x \\ &\quad -(9x^2 - 9x) \\ &\qquad 8x - 8 \\ &\qquad -(8x - 8) \\ &\qquad\quad 0 \end{aligned}$$

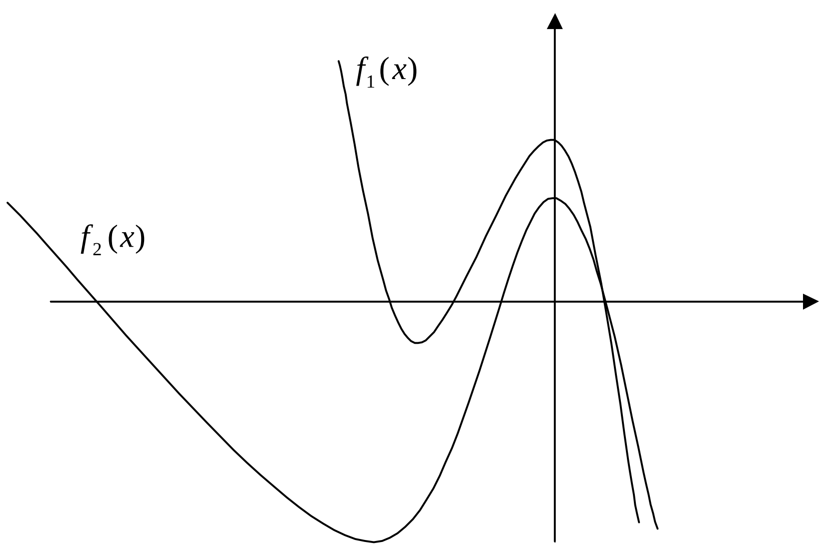
$x^2 + 9x + 8 = 0$ p-q-Formel

$x_{2/3} = -4{,}5 \pm \sqrt{4{,}5^2 - 8}$

$x_2 = -1$

$x_3 = -8$

$S_{x1}(1|0)$ $S_{x2}(-1|0)$ $S_{x3}(-8|0)$



gemeinsamer Schnittpunkt bei S(1l0)

$f_2(x) = f_2(x)$

$-0{,}5x^3 - 2x^2 - 0{,}5x + 3 = -0{,}25x^3 - 2x^2 + 0{,}25x + 2 \quad | + 0{,}25x^3 + 2x^2 - 0{,}25x - 2$

$-0{,}25x^3 - 0{,}75x + 1 = 0 \quad | : (-0{,}25)$

$x^3 + 3x - 4 = 0$

$x_1 = 1$ Schnittpunkt !

$(x^3 + 0x^2 + 3x - 4) : (x - 1) = x^2 + x + 4$

$-(x^3 - x^2)$

$x^2 + 3x$

$-(x^2 - x)$

$4x - 4$

$-(4x - 4)$

0

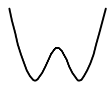
$x^2 + x + 4 = 0$

$x_{2/3} = -0{,}5 \pm \sqrt{0{,}5^2 - 4}$

$n.l.$

Es gibt nur den Schnittpunkt S(1l0).

<u>2. Aufgabe</u>

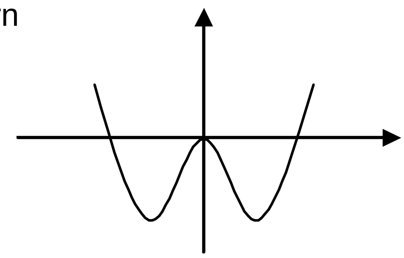
$f_1(x) = x^4 - 9x^2$ 1. $D = R$ 2. $x \to -\infty;\ f(x) \to +\infty$; $x \to +\infty;\ f(x) \to +\infty$  3. AS

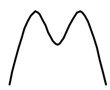
4. $S_y(0|0)$ $f(x) = 0$ $0 = x^4 - 9x^2$ ausklammern

$0 = x^2(x^2 - 9)$

$x_{1/2} = 0$; $x_3 = 3$; $x_4 = -3$

$S_{x1/2}(0|0)$ $S_{x3}(3|0)$ $S_{x4}(-3|0)$



$f_2(x) = -0{,}25x^4 + 2x^2 - 4$ 1. $D = R$ 2. $x \to -\infty;\ f(x) \to -\infty$; $x \to +\infty;\ f(x) \to -\infty$  3. AS

4. $S_y(0|-4)$ $f(x) = 0$ $0 = -0{,}25x^4 + 2x^2 - 4 | : (-0{,}25)$

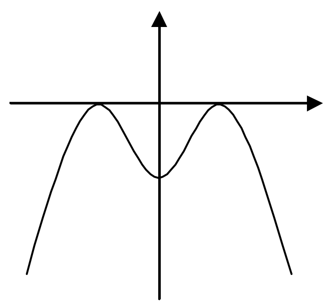
$0 = x^4 - 8x^2 + 16$ Substitution mit $x^2 = z$ p-q-Formel

$z_{1/2} = 4$ Resubstitution mit $z = x^2$ ergibt

$x^2 = 4$ und noch mal $x^2 = 4$

=> $x_{1/2} = 2$ und $x_{1/2} = -2$

$S_{x1/2}(2|0)$ $S_{x1/2}(-2|0)$



<u>3. Aufgabe</u>

a) $f_1(x) = \frac{2x - 5}{x + 2}$

1.

$N(x) = 0$

$0 = x + 2 \quad | -2$ $D = R \setminus \{-2\}$

$-2 = x$

5. Zg=Ng Polynomdivision

$(2x - 5) : (x + 2) = 2 - \frac{9}{x + 2}$

$-(2x + 4)$

-9

$y_A = 2$

2. $f(0) = -2{,}5 \qquad S_y(0|-2{,}5)$

$f(x) = 0$

$2x - 5 = 0 \qquad S_x(2{,}5|0)$

$x = 2{,}5$

3. keine behebbare Lücke

4. $x = -2$ ist Pol

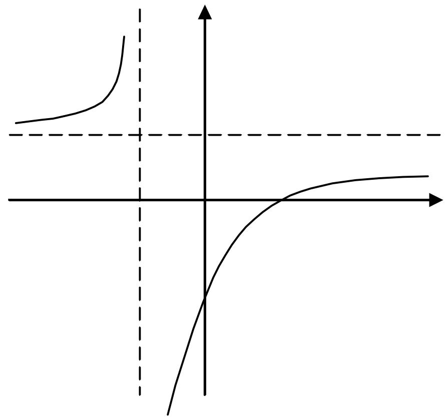
$\left.\begin{array}{l} l-\lim\limits_{x\to -2} f_1(x) = +\infty \\ r-\lim\limits_{x\to -2} f_1(x) = -\infty \end{array}\right\}$ Pol mit VZW

$x \to -\infty;\ R(x) < 0 \quad von \quad unten$

$x \to +\infty;\ R(x) > 0 \quad von \quad oben$

6. KS

7. Zeichnung



b) $f_2(x) = \dfrac{9}{x-3}$

1.
$N(x) = 0$

$0 = x - 3$

$x = 3$

$D = R \setminus \{3\}$

2. $f(0) = -3 \qquad S_y(0|-3)$

$f(x) = 0$

$9 \neq 0$ kein S_x

3. keine behebbare Lücke

4. $x = 3$ ist Pol

$\left.\begin{array}{l} l-\lim\limits_{x\to 3} f_2(x) = -\infty \\ r-\lim\limits_{x\to 3} f_2(x) = +\infty \end{array}\right\}$ Pol mit VZW

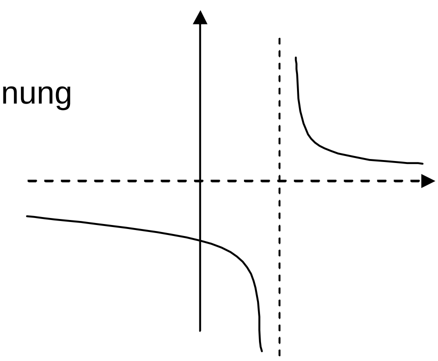
5. Zg<Ng => $y_A = 0$

$x \to -\infty;\ R(x) < 0 \quad von \quad unten$

$x \to +\infty;\ R(x) > 0 \quad von \quad oben$

6. KS

7. Zeichnung



c) $f_3(x) = \dfrac{x^2 + 2x - 3}{x^2 - x}$

1. $N(x) = 0$

$0 = x^2 - x$

$0 = x(x-1)$

$x_1 = 0;\ x_2 = 1 \quad D = R / \{0;1\}$

2. $f(0) = n.l.$

$f(x) = 0$

$0 = x^2 + 2x - 3$ pq

$x_1 = 1$ => kein S_x

$x_2 = -3$ => $S_x(-3|0)$

5. Zg = Ng => Polynomdivision mit f*(x)

$(x+3) : x = 1 + \dfrac{3}{x}$

$\underline{-(x)\qquad}$

$+3$

$y_A = 1$

$x \to -\infty;\ R(x) < 0 \quad von \quad unten$

$x \to +\infty;\ R(x) > 0 \quad von \quad oben$

6. KS

3. $x = 1$ ist behebbare Lücke

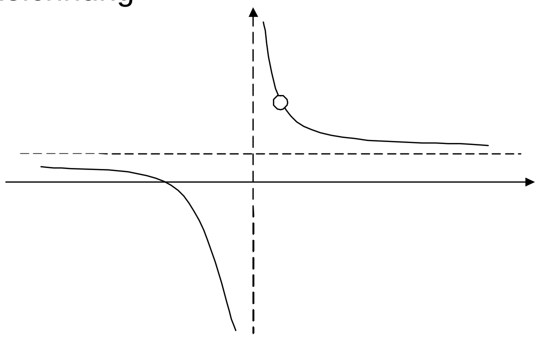
$$f(x) = \frac{(x-1)(x+3)}{x(x-1)}$$

$$f^*(x) = \frac{x+3}{x}$$

4. $x = 0$ ist Pol

$$\left.\begin{array}{l} l-\lim\limits_{x\to 0} f(x) = -\infty \\ r-\lim\limits_{x\to 0} f(x) = +\infty \end{array}\right\}$$ Pol mit VZW

7. Zeichnung



<u>4. Aufgabe</u>

a) $f_1(x) = g(x)$

$\frac{2x-5}{x+2} = 3 \,|\cdot (x+2)$

$2x - 5 = 3(x+2)$

$2x - 5 = 3x + 6$

$\Rightarrow x = -11$

$g(-11) = 3$ bzw. $f_1(-11) = 3$

$S(-11|3)$

b) $f_2(x) = g(x)$

$\frac{9}{x-3} = 3 \,|\cdot (x-3)$

$9 = 3(x-3)$

$9 = 3x - 9$

$\Rightarrow x = 6$

$g(6) = 3$ bzw. $f_2(6) = 3$

$S(6|3)$

c) $f_3(x) = g(x)$

$\frac{x^2+2x-3}{x^2-x} = 3 \,|\cdot (x^2-x)$

$x^2 + 2x - 3 = 3(x^2 - x)$

$x^2 + 2x - 3 = 3x^2 - 3x$

$0 = 2x^2 - 5x + 3$

$0 = x^2 - 2{,}5x + 1{,}5$ pq

$x_1 = 1{,}5$; $x_2 = 1$ => kein Schnittpunkt, da behebbare Lücke !!!

$f_3(1{,}5) = 3$

$S(1{,}5|3)$