Lösungen C t13

1. Aufgabe

1)

1. $D = R$ $\qquad$ $f(x) = \frac{1}{2}x^3 + 2x^2 - \frac{1}{2}x - 2$

2.  $\qquad$ $f'(x) = \frac{3}{2}x^2 + 4x - \frac{1}{2}$

3. KS $\qquad$ $f''(x) = 3x + 4$

4. $S_y(0|-2)$ $\qquad$ $f'''(x) = 3$

$$f(x) = 0$$
$$0 = \frac{1}{2}x^3 + 2x^2 - \frac{1}{2}x - 2 \,\Big|\, : \frac{1}{2}$$
$0 = x^3 + 4x^2 - x - 4$ Polynomdivision mit $x_1 = 1$ führt zu

$0 = x^2 + 5x + 4$ p-q-Formel ergibt $x_2 = -1$ und $x_3 = -4$

$S_{x1}(1|0)$ $\quad$ $S_{x2}(-1|0)$ $\quad$ $S_{x3}(-4|0)$

5. Extrempunkte

$f'(x) = 0$ $\qquad$ $f'(x) = 0 \ \wedge \ f''(x) \neq 0$

$0 = \frac{3}{2}x^2 + 4x - \frac{1}{2} \,\Big|\, : \frac{3}{2}$ $\qquad$ $f''(0{,}1) = 4{,}3 > 0 \Rightarrow T$

$0 = x^2 + \frac{8}{3}x - \frac{1}{3}$ $\qquad$ $f''(-2{,}8) = -4{,}4 < 0 \Rightarrow H$

$x_{1/2} = -\frac{4}{3} \pm \sqrt{\left(\frac{4}{3}\right)^2 + \frac{1}{3}}$ $\qquad$ $f(0{,}1) = -2{,}0$ $\qquad$ $T(0{,}1|-2)$

$x_1 = 0{,}1 \vee x_2 = -2{,}8$ $\qquad$ $f(-2{,}8) = 4{,}1$ $\qquad$ $H(-2{,}8|4{,}1)$

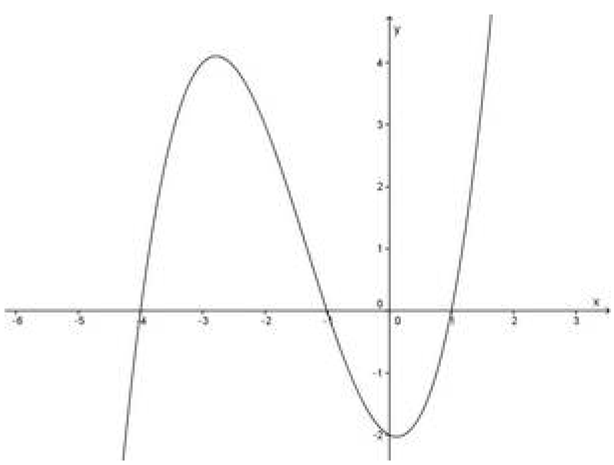
6. Wendepunkte

$f''(x) = 0$ $\qquad$ $f''(x) = 0 \ \wedge \ f'''(x) \neq 0$

$0 = 3x + 4$ $\qquad$ $f'''(-1{,}3) = 3 > 0 \Rightarrow R - L - K$

$x = -1{,}3$ $\qquad$ $f(-1{,}3) = 0{,}9$ $\qquad$ $W_{R-L}(-1{,}3|0{,}9)$

7. Zeichnung



2)

1. $D = R$

2.

3. PS

4. $S_y(0|0)$

$f(x) = -0{,}1x^3 + 0{,}9x$

$f'(x) = -0{,}3x^2 + 0{,}9$

$f''(x) = -0{,}6x$

$f'''(x) = -0{,}6$

$f(x) = 0$

$0 = -0{,}1x^3 + 0{,}9x \,|: (-0{,}1)$

$0 = x^3 - 9x$ x ausklammern mit $x_1 = 0$ führt zu

$0 = x^2 - 9$ Wurzel ziehen ergibt $x_2 = 3$ und $x_3 = -3$

$S_{x1}(0|0)$ $S_{x2}(3|0)$ $S_{x3}(-3|0)$

5. Extrempunkte

$f'(x) = 0$

$0 = -0{,}3x^2 + 0{,}9 \,|: (-0{,}3)$

$0 = x^2 - 3$

$x^2 = 3 \,|\sqrt{}$

$x_1 = 1{,}7 \lor x_2 = -1{,}7$

$f'(x) = 0 \land f''(x) \neq 0$

$f''(1{,}7) = -1{,}0 < 0 \Rightarrow H$

$f''(-1{,}7) = 1{,}0 > 0 \Rightarrow T$

$f(1{,}7) = 1{,}0$ $H(1{,}7|1{,}0)$

$f(-1{,}7) = -1{,}0$ $T(-1{,}7|-1{,}0)$

6. Wendepunkte

$f''(x) = 0$

$0 = -0{,}6x$

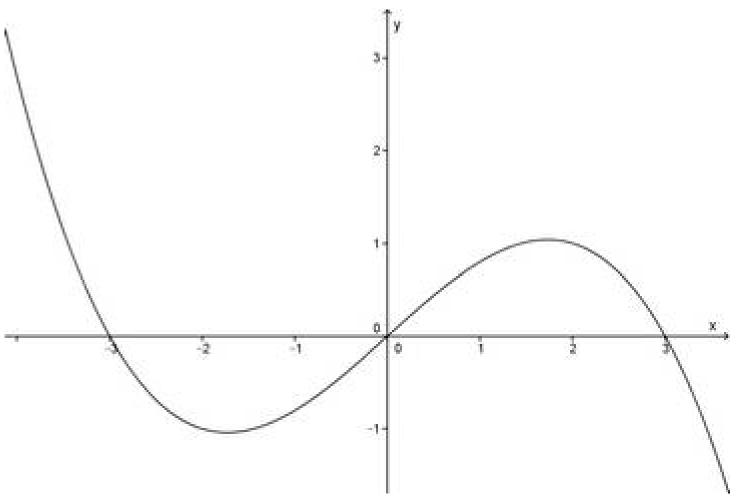
$x = 0$

$f''(x) = 0 \land f'''(x) \neq 0$

$f'''(0) = -0{,}6 < 0 \Rightarrow L-R-K$

$f(0) = 0$ $W_{L-R}(0|0)$

7. Zeichnung



3)

1. $D = R$ $\qquad$ $f(x) = -0{,}2x^4 + 2x^2 - 1{,}8$

2.  $\qquad$ $f'(x) = -0{,}8x^3 + 4x$

3. AS $\qquad$ $f''(x) = -2{,}4x^2 + 4$

4. $S_y(0|-1{,}8)$ $\qquad$ $f'''(x) = -4{,}8x$

$f(x) = 0$

$0 = -0{,}2x^4 + 2x^2 - 1{,}8 |:(-0{,}2)$

$0 = x^4 - 10x^2 + 9$ Substitution mit $x^2 = z$ führt zu

$0 = z^2 - 10z + 9$ p-q-Formel ergibt $z_1 = 9$ und $z_2 = 1$

Resubstitution mit $z = x^2$ liefert $x^2 = 9$ und $x^2 = 1$; Wurzel ziehen ergibt dann

$x_1 = 3$ V $x_2 = -3$ V $x_3 = 1$ V $x_4 = -1$

$S_{x1}(3|0)$ $\quad$ $S_{x2}(-3|0)$ $\quad$ $S_{x3}(1|0)$ $\quad$ $S_{x4}(-1|0)$

5. Extrempunkte

$f'(x) = 0$ $\qquad$ $f'(x) = 0 \ \wedge \ f''(x) \neq 0$

$0 = -0{,}8x^3 + 4x |:(-0{,}8)$ $\qquad$ $f''(0) = 4 > 0 \Rightarrow T$

$0 = x^3 - 5x$ x ausklammern $\qquad$ $f''(2{,}2) = -7{,}6 < 0 \Rightarrow H$

$x_1 = 0 \ \wedge \ x^2 - 5 = 0$ $\qquad$ $f''(-2{,}2) = -7{,}6 < 0 \Rightarrow H$

$x^2 = 5 |\sqrt{\ }$ $\qquad$ $f(2{,}2) = 3{,}2$ $\qquad$ $H_1(2{,}2|3{,}2)$

$x_2 = 2{,}2$ V $x_2 = -2{,}2$ $\qquad$ $f(-2{,}2) = 3{,}2$ $\qquad$ $H_2(-2{,}2|3{,}2)$

$f(0) = -1{,}8$ $\qquad$ $T(0|-1{,}8)$

6. Wendepunkte

$f''(x) = 0$ $\qquad$ $f''(x) = 0 \ \wedge \ f'''(x) \neq 0$

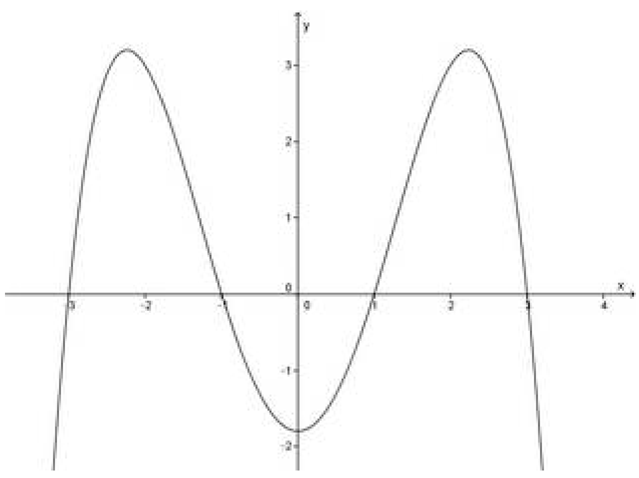
$0 = -2{,}4x^2 + 4 |:(-2{,}4)$ $\qquad$ $f'''(1{,}3) = -6{,}2 < 0 \Rightarrow L - R - K$

$0 = x^2 - \frac{5}{3}$ $\qquad$ $f'''(-1{,}3) = 6{,}2 > 0 \Rightarrow R - L - K$

$x^2 = \frac{5}{3} |\sqrt{\ }$ $\qquad$ $f(1{,}3) = 1{,}0$ $\qquad$ $W_{L-R}(1{,}3|1)$

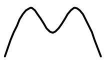
$x_1 = 1{,}3$ V $x_2 = -1{,}3$ $\qquad$ $f(-1{,}3) = 1{,}0$ $\qquad$ $W_{R-L}(-1{,}3|1)$

7. Zeichnung



4)

1. $D = R$

2. 

3. AS

4. $S_y(0|2)$

$f(x) = -0{,}5x^4 + 1{,}5x^2 + 2$

$f'(x) = -2x^3 + 3x$

$f''(x) = -6x^2 + 3$

$f'''(x) = -12x$

$f(x) = 0$

$0 = -0{,}5x^4 + 1{,}5x^2 + 2 \,|: (-0{,}5)$

$0 = x^4 - 3x^2 - 4$ Substitution mit $x^2 = z$ führt zu

$0 = z^2 - 3z - 4$ p-q-Formel ergibt $z_1 = 4$ und $z_2 = -1$

Resubstitution mit $z = x^2$ liefert $x^2 = 4$ und $x^2 = -1$; Wurzel ziehen ergibt dann

$x_1 = 2 \lor x_2 = -2 \lor x_{3/4} = n.l.$

$S_{x1}(2|0)$ $S_{x2}(-2|0)$

5. Extrempunkte

$f'(x) = 0$

$0 = -2x^3 + 3x \,|: (-2)$

$0 = x^3 - 1{,}5x$ x ausklammern

$x_1 = 0 \land x^2 - 1{,}5 = 0$

$x^2 = 1{,}5 \,|\sqrt{\ }$

$x_2 = 1{,}2 \lor x_2 = -1{,}2$

$f'(x) = 0 \land f''(x) \neq 0$

$f''(0) = 3 > 0 \Rightarrow T$

$f''(1{,}2) = -5{,}6 < 0 \Rightarrow H$

$f''(-1{,}2) = -5{,}6 < 0 \Rightarrow H$

$f(1{,}2) = 3{,}1$ $H_1(1{,}2|3{,}1)$

$f(-1{,}2) = 3{,}1$ $H_2(-1{,}2|3{,}1)$

$f(0) = 2$ $T(0|2)$

6. Wendepunkte

$f''(x) = 0$

$0 = -6x^2 + 3 \,|: (-6)$

$0 = x^2 - 0{,}5$

$x^2 = 0{,}5 \,|\sqrt{\ }$

$x_1 = 0{,}7 \lor x_2 = -0{,}7$

$f''(x) = 0 \land f'''(x) \neq 0$

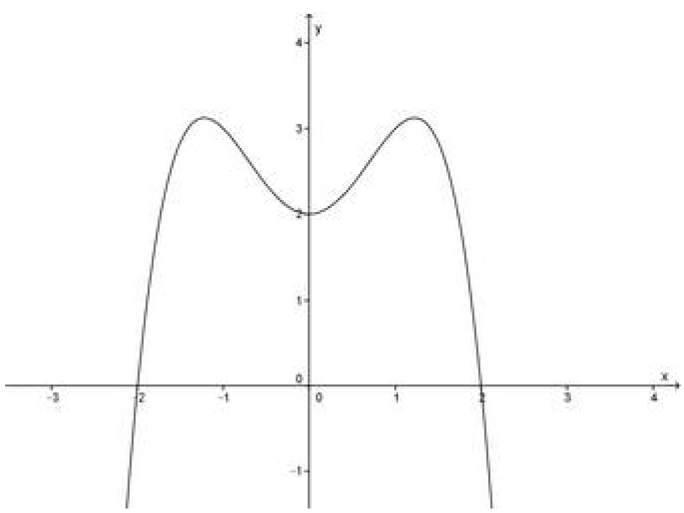
$f'''(0{,}7) = -8{,}4 < 0 \Rightarrow L - R - K$

$f'''(-0{,}7) = 8{,}4 > 0 \Rightarrow R - L - K$

$f(0{,}7) = 2{,}6$ $W_{L-R}(0{,}7|2{,}6)$

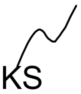
$f(-0{,}7) = 2{,}6$ $W_{R-L}(-0{,}7|2{,}6)$

7. Zeichnung



5)

1. $D = R$

2. 

3. KS

4. $S_y(0|0)$

$f(x) = 2x^3 - 8x^2 + 8x$

$f'(x) = 6x^2 - 16x + 8$

$f''(x) = 12x - 16$

$f'''(x) = 12$

$f(x) = 0$

$0 = 2x^3 - 8x^2 + 8x \,|:2$

$0 = x^3 - 4x^2 + 4x$ ausklammern mit $x_1 = 0$ führt zu

$0 = x^2 - 4x + 4$ p-q-Formel ergibt $x_{2/3} = 2$

$S_{x1}(0|0) \qquad S_{x2/3}(2|0)$

5. Extrempunkte

$f'(x) = 0$

$0 = 6x^2 - 16x + 8 \,|:6$

$0 = x^2 - \frac{8}{3}x + \frac{4}{3}$

$x_{1/2} = +\frac{4}{3} \pm \sqrt{\left(\frac{4}{3}\right)^2 - \frac{4}{3}}$

$x_1 = 2 \lor x_2 = 0{,}7$

$f'(x) = 0 \;\land\; f''(x) \neq 0$

$f''(2) = 8 > 0 \Rightarrow T$

$f''(0{,}7) = -7{,}6 < 0 \Rightarrow H$

$f(2) = 0 \qquad T(2|0)$

$f(0{,}7) = 2{,}4 \qquad H(0{,}7|2{,}4)$

6. Wendepunkte

$f''(x) = 0$

$0 = 12x - 16$

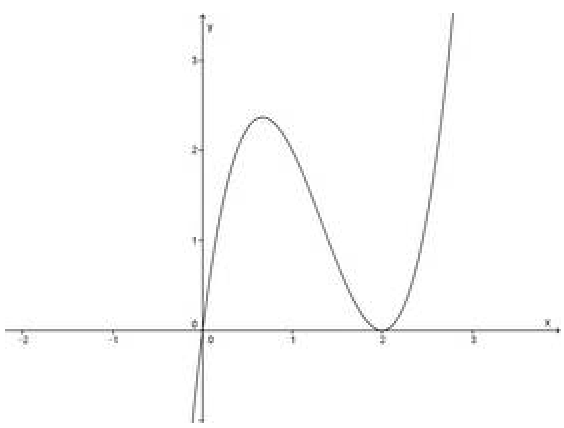
$x = 1{,}3$

$f''(x) = 0 \;\land\; f'''(x) \neq 0$

$f'''(1{,}3) = 12 > 0 \Rightarrow R-L-K$

$f(1{,}3) = 1{,}3 \qquad W_{R-L}(1{,}3|1{,}3)$

7. Zeichnung



6)

1. $D = R$ $\qquad f(x) = \frac{1}{10}x^3 - \frac{21}{10}x - 2$

2.  $\qquad f'(x) = \frac{3}{10}x^2 - \frac{21}{10}$

3. KS $\qquad f''(x) = \frac{3}{5}x$

4. $S_y(0|-2)$ $\qquad f'''(x) = \frac{3}{5}$

$f(x) = 0$

$f(x) = \frac{1}{10}x^3 - \frac{21}{10}x - 2 \,|: 0{,}1$

$0 = x^3 - 21x - 20$ Polynomdivision mit $x_1 = -1$ führt zu

$0 = x^2 - x - 20$ p-q-Formel ergibt $x_2 = 5$ V $x_3 = -4$

$S_{x1}(-1|0) \quad S_{x2}(5|0) \quad S_{x3}(-4|0)$

5. Extrempunkte

$f'(x) = 0$ $\qquad f'(x) = 0 \;\wedge\; f''(x) \neq 0$

$f'(x) = \frac{3}{10}x^2 - \frac{21}{10} \,|: 0{,}3$ $\qquad f''(2{,}6) = 1{,}6 > 0 \Rightarrow T$

$0 = x^2 - 7$ $\qquad f''(-2{,}6) = -1{,}6 < 0 \Rightarrow H$

$x^2 = 7 \,|\sqrt{}$ $\qquad f(2{,}6) = -5{,}7$ $\qquad T(2{,}6|-5{,}7)$

$x_1 = 2{,}6$ V $x_2 = -2{,}6$ $\qquad f(-2{,}6) = 1{,}7$ $\qquad H(-2{,}6|1{,}7)$

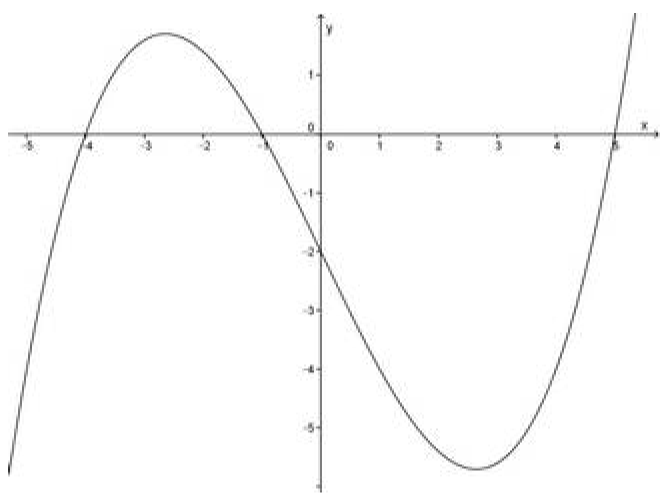
6. Wendepunkte

$f''(x) = 0$ $\qquad f''(x) = 0 \;\wedge\; f'''(x) \neq 0$

$0 = \frac{3}{5}x$ $\qquad f'''(0) = 0{,}6 > 0 \Rightarrow R - L - K$

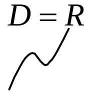
$x = 0$ $\qquad f(0) = -2$ $\qquad W_{R-L}(0|-2)$

7. Zeichnung



7)

1. $D = R$

2. 

3. KS

4. $S_y(0|-1)$

$f(x) = 0{,}5x^3 + 1{,}5x^2 - x - 1$

$f'(x) = 1{,}5x^2 + 3x - 1$

$f''(x) = 3x + 3$

$f'''(x) = 3$

$f(x) = 0$

$0 = 0{,}5x^3 + 1{,}5x^2 - x - 1 \,|: 0{,}5$

$0 = x^3 + 3x^2 - 2x - 2$ Polynomdivision mit $x_1 = 1$ führt zu

$0 = x^2 + 4x + 2$ p-q-Formel ergibt $x_2 = -0{,}6$ und $x_3 = -3{,}4$

$S_{x1}(1|0)$ $S_{x2}(-0{,}6|0)$ $S_{x3}(-3{,}4|0)$

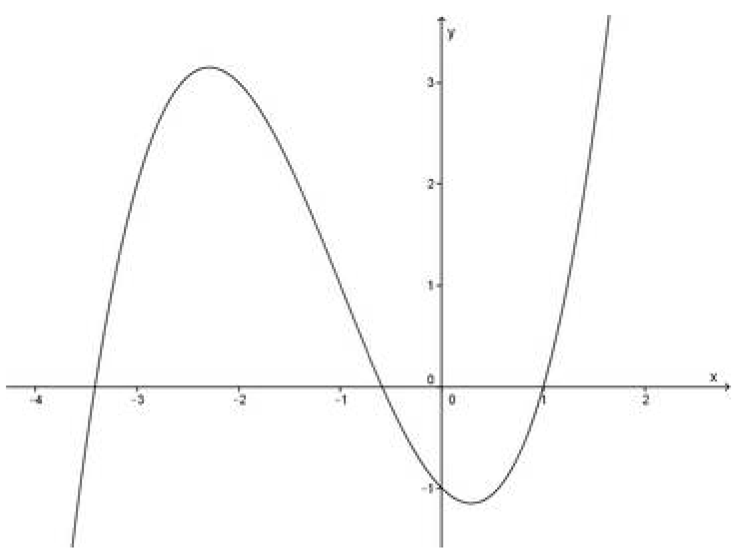
5. Extrempunkte

$f'(x) = 0$	$f'(x) = 0 \;\wedge\; f''(x) \neq 0$	
$0 = 1{,}5x^2 + 3x - 1 \,|: 1{,}5$	$f''(0{,}3) = 3{,}9 > 0 \Rightarrow T$	
$0 = x^2 + 2x - \frac{2}{3}$	$f''(-2{,}3) = -3{,}9 < 0 \Rightarrow H$	
$x_{1/2} = -1 \pm \sqrt{1 + \frac{2}{3}}$	$f(0{,}3) = -1{,}2$	$T(0{,}3|-1{,}2)$
$x_1 = 0{,}3$ V $x_2 = -2{,}3$	$f(-2{,}3) = 3{,}2$	$H(-2{,}3|3{,}2)$

6. Wendepunkte

$f''(x) = 0$	$f''(x) = 0 \;\wedge\; f'''(x) \neq 0$	
$0 = 3x + 3$	$f'''(-1) = 3 > 0 \Rightarrow R - L - K$	
$x = -1$	$f(-1) = 1$	$W_{R-L}(-1|1)$

7. Zeichnung



8)

1. $D = R$

2. 

3. AS

4. $S_y(0|-3)$

$f(x) = -0{,}5x^4 + 2x^2 - 3$

$f'(x) = -2x^3 + 4x$

$f''(x) = -6x^2 + 4$

$f'''(x) = -12x$

$f(x) = 0$

$0 = -0{,}5x^4 + 2x^2 - 3 \,|: (-0{,}5)$

$0 = x^4 - 4x^2 + 6$ Substitution mit $x^2 = z$ führt zu

$0 = z^2 - 4z + 6$ p-q-Formel ergibt n.l. da Wurzel negativ

=> keine Nullstellen

5. Extrempunkte

$f'(x) = 0$

$0 = -2x^3 + 4x \,|: (-2)$

$0 = x^3 - 2x$ x ausklammern

$x_1 = 0 \;\wedge\; x^2 - 2 = 0$

$x^2 = 2 \,|\sqrt{\;}$

$x_2 = 1{,}4 \;\vee\; x_2 = -1{,}4$

$f'(x) = 0 \;\wedge\; f''(x) \neq 0$

$f''(0) = 4 > 0 \Rightarrow T$

$f''(1{,}4) = -7{,}8 < 0 \Rightarrow H$

$f''(-1{,}4) = -7{,}8 < 0 \Rightarrow H$

$f(1{,}4) = -1{,}0$ $H_1(1{,}4|-1)$

$f(-1{,}4) = -1{,}0$ $H_2(-1{,}4|-1)$

$f(0) = -3$ $T(0|-3)$

6. Wendepunkte

$f''(x) = 0$

$0 = -6x^2 + 4 \,|: (-6)$

$0 = x^2 - \frac{2}{3}$

$x^2 = \frac{2}{3} \,|\sqrt{\;}$

$x_1 = 0{,}8 \;\vee\; x_2 = -0{,}8$

$f''(x) = 0 \;\wedge\; f'''(x) \neq 0$

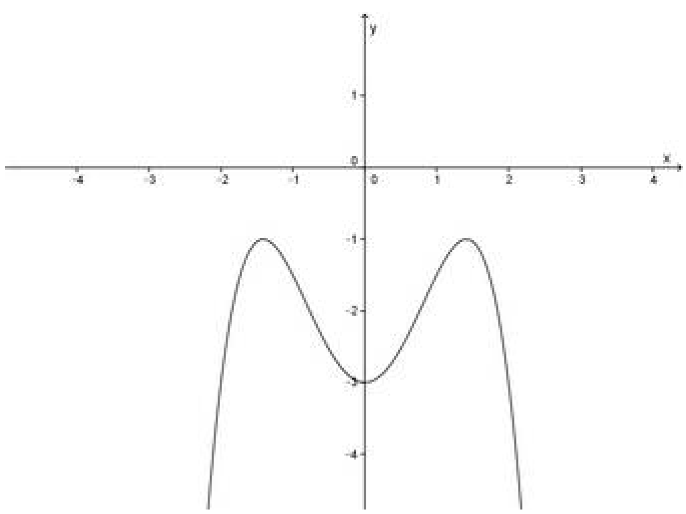
$f'''(0{,}8) = -9{,}6 < 0 \Rightarrow L - R - K$

$f'''(-0{,}8) = 9{,}6 > 0 \Rightarrow R - L - K$

$f(0{,}8) = -1{,}9$ $W_{L-R}(0{,}8|-1{,}9)$

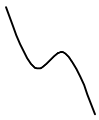
$f(-0{,}8) = -1{,}9$ $W_{R-L}(-0{,}8|-1{,}9)$

7. Zeichnung



2. Aufgabe
a)

1. $D=R$ $\qquad$ $f(x)=-\frac{1}{4}x^3+3x$

2.  $\qquad$ $f'(x)=-\frac{3}{4}x^2+3$

3. PS $\qquad$ $f''(x)=-\frac{3}{2}x$

4. $S_y(0|0)$ $\qquad$ $f'''(x)=-\frac{3}{2}$

$f(x)=0$

$0=-\frac{1}{4}x^3+3x \,\Big|\, :\left(-\frac{1}{4}\right)$

$0=x^3-12x$ $\qquad$ ausklammern mit $x_1=0$ führt zu

$0=x^2-12$ $\qquad$ umformen und Wurzel ziehen ergibt $x_2=3{,}5$ und $x_3=-3{,}5$

$S_{x1}(0|0)$ $\qquad$ $S_{x2}(3{,}5|0)$ $\qquad$ $S_{x3}(-3{,}5|0)$

5. Extrempunkte

$f'(x)=0$ $\qquad$ $f'(x)=0 \;\wedge\; f''(x)\neq 0$

$0=-\frac{3}{4}x^2+3 \,\Big|\, :\left(-\frac{3}{4}\right)$ $\qquad$ $f''(2)=-3<0 \Rightarrow H$

$0=x^2-4$ $\qquad$ $f''(-2)=3>0 \Rightarrow T$

$x^2=4 \,\big|\, \sqrt{}$ $\qquad$ $f(2)=4$ $\qquad$ $H(2|4)$

$x_1=2 \vee x_2=-2$ $\qquad$ $f(-2)=-4$ $\qquad$ $T(-2|-4)$

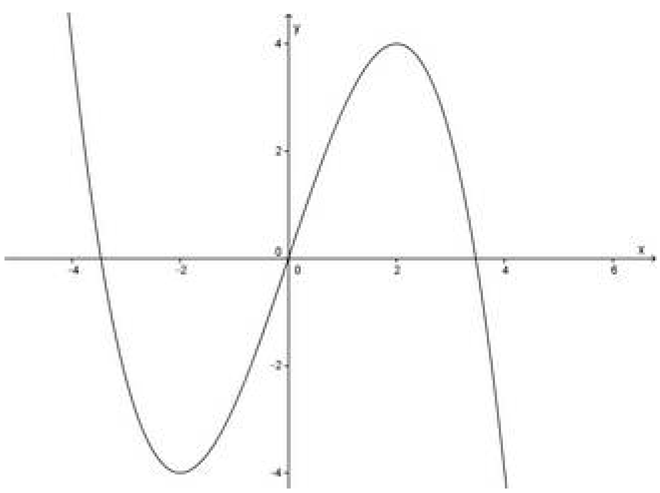
6. Wendepunkte

$f''(x)=0$ $\qquad$ $f''(x)=0 \;\wedge\; f'''(x)\neq 0$

$0=-\frac{3}{2}x \,\Big|\, :\left(-\frac{3}{2}\right)$ $\qquad$ $f'''(0)=-1{,}5<0 \Rightarrow L-R-K$

$x=0$ $\qquad$ $f(0)=0$ $\qquad$ $W_{L-R}(0|0)$

7. Zeichnung



b) $f'(x) = m$ $\quad f'(x) = -\frac{3}{4}x^2 + 3$ mit $x_1 = -4$ ergibt sich $f'(-4) = -9$ also $m_1 = -9$

c) $f'(x) = m$ $\quad f'(x) = -\frac{3}{4}x^2 + 3$ mit $x_2 = +1$ ergibt sich $f'(1) = 2{,}25$ also $m_2 = 2{,}25$

d) $f'(x) = m$ $\quad f'(x) = -\frac{3}{4}x^2 + 3$ mit $m = -\frac{15}{4}$ ergibt sich

$$-\frac{15}{4} = -\frac{3}{4}x^2 + 3 \quad \Big| -3$$

$$-\frac{27}{4} = -\frac{3}{4}x^2 \quad \Big| : \left(-\frac{3}{4}\right)$$

Stellen = x-Werte

$$9 = x^2 \quad \Big|\sqrt{}$$

daraus ergibt sich $x_1 = 3$ V $x_2 = -3$

e) $f'(x) = m$ $\quad f'(x) = -\frac{3}{4}x^2 + 3$ mit $m = 3$ ergibt sich

$$3 = -\frac{3}{4}x^2 + 3 \quad \Big| -3$$

$$0 = -\frac{3}{4}x^2 \quad \Big| : \left(-\frac{3}{4}\right)$$

$$0 = x^2 \quad \Big|\sqrt{}$$

daraus ergibt sich $x_{1/2} = 0$

f) $t(x) = m \cdot x + b$ => y-Wert zu x = 0 in der Ausgangsfunktion berechnen

$f(0) = 0$ mit m = 3 ergibt sich

$$0 = 3 \cdot 0 + b$$
$$b = 0$$

=> $t(x) = 3x$

g) senkrecht = orthogonal => $m_1 \cdot m_2 = -1$

$m_1 = 3$ => $m_2 = -\frac{1}{3}$ Da es um den Ursprung geht, bleibt b = 0.

$n(x) = -\frac{1}{3}x$ Gleichung der Normalen

3. Aufgabe

$x = 4$ einsetzen in die Tangentengleichung, da Berührpunkt gemeinsamer Punkt von f(x) und t(x) ist

$t(4) = -9 \cdot 4 + 32 = -4 \Rightarrow P(4|-4)$

Punkt P einsetzen in $f(x) = ax^3 + 3x$ um a zu berechnen

$$-4 = a \cdot 4^3 + 3 \cdot 4$$
$$-4 = 64a + 12$$
$$-16 = 64a$$
$$a = -0{,}25$$
$$f(x) = 0{,}25x^3 + 3x$$