Lösungen B 16

1. Aufgabe

a) $f(x) = \frac{1}{4}x^3 + x^2 - \frac{1}{4}x - 1$ 1. $D = R$ 2. $x \to -\infty; f(x) \to -\infty$ / $x \to +\infty; f(x) \to +\infty$  3. KS

4. $S_y(0|-1)$ $f(x) = 0$ $0 = \frac{1}{4}x^3 + x^2 - \frac{1}{4}x - 1 \;\Big|\; : \frac{1}{4}$

$0 = x^3 + 4x^2 - x - 4$ Polynomdivision mit $x_1 = 1$

$$(x^3 + 4x^2 - x - 4) : (x - 1) = x^2 + 5x + 4$$

$-(x^3 - 1x^2)$

$5x^2 - x$

$-(5x^2 - 5x)$

$4x - 4$

$-(4x - 4)$

0

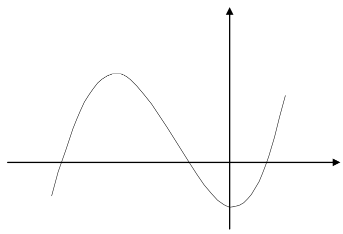
$x^2 + 5x + 4 = 0$

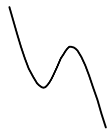
$x_{2/3} = -2{,}5 \pm \sqrt{2{,}5^2 - 4}$

$x_2 = -1$

$x_3 = -4$

$S_{x1}(1|0)$ $S_{x2}(-1|0)$ $S_{x3}(-4|0)$

5. 

b) $f(x) = -0{,}5x^3 + 4{,}5x$ 1. $D = R$ 2. $x \to -\infty; f(x) \to +\infty$ / $x \to +\infty; f(x) \to -\infty$  3. PS

4. $S_y(0|0)$ $f(x) = 0$ $0 = -0{,}5x^3 + 4{,}5x \;\Big|\; : (-0{,}5)$

$0 = x^3 - 9x$ x ausklammern

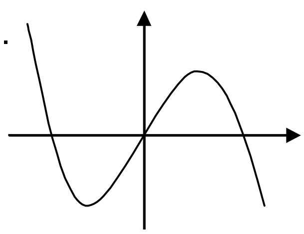
$0 = x(x^2 - 9)$

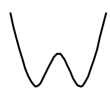
$x_1 = 0$; $x^2 - 9 = 0 \;|\; +9$

$x^2 = 9 \;|\sqrt{\ }$

$x_2 = 3$; $x_3 = -3$

$S_{x1}(0|0)$ $S_{x2}(3|0)$ $S_{x3}(-3|0)$

5. 

c) $f(x) = \frac{1}{4}x^4 - \frac{7}{4}x^3 + 3x^2 + x - 4$ 1. $D = R$ 2. $x \to -\infty; f(x) \to +\infty$ / $x \to +\infty; f(x) \to +\infty$  3. KS

4. $S_y(0|-4)$ $f(x) = 0$ $0 = \frac{1}{4}x^4 - \frac{7}{4}x^3 + 3x^2 + x - 4 \;\Big|\; : \frac{1}{4}$

$0 = x^4 - 7x^3 + 12x^2 + 4x - 16$ Polynomdivision mit $x_1 = -1$

$$(x^4 - 7x^3 + 12x^2 + 4x - 16) : (x + 1) = x^3 - 8x^2 + 20x - 16$$

$$-(x^4 + x^3)$$

$$-8x^3 + 12x^2$$

$$-(-8x^3 - 8x^2)$$

$$20x^2 + 4x$$

$$-(20x^2 + 20x)$$

$$-16x - 16$$

$$-(-16x - 16)$$

$$0$$

$x^3 - 8x^2 + 20x - 16 = 0$ Polynomdivision mit $x_2 = 2$

$$(x^3 - 8x^2 + 20x - 16) : (x - 2) = x^2 - 6x + 8$$

$$-(x^3 - 2x^2)$$

$$-6x^2 + 20x$$

$$-(-6x^2 + 12x)$$

$$8x - 16$$

$$-(8x - 16)$$

$$0$$

$x^2 - 6x + 8 = 0$

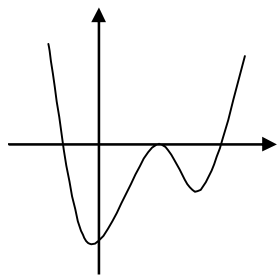
$x_{3/4} = +3 \pm \sqrt{9 - 8}$

$x_3 = 4$

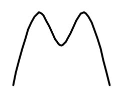
$x_4 = 2$ doppelte Lösung

$S_{x1}(-1|0)$ $S_{x2/4}(2|0)$ $S_{x3}(4|0)$

5.



d) $f(x) = -0{,}1x^4 + x^2 - 0{,}9$ 1. $D = R$ 2. $x \to -\infty; f(x) \to -\infty$; $x \to +\infty; f(x) \to -\infty$



3. AS

4. $S_y(0|-0{,}9)$ $f(x) = 0$ $0 = -0{,}1x^4 + x^2 - 0{,}9 \; | : (-0{,}1)$

$0 = x^4 - 10x^2 + 9$ biquadratische Gleichung, Substitution

$x^2 = z$

$0 = z^2 - 10z + 9$ p-q-Formel

$z_{1/2} = +5 \pm \sqrt{5^2 - 9}$

$z_1 = 9$

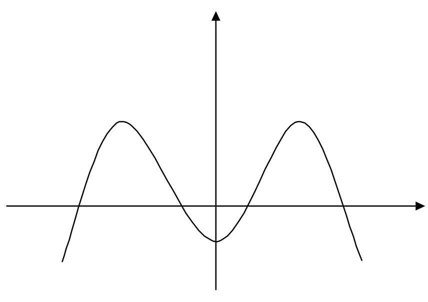
$z_2 = 1$

$z = x^2$ Resubstitution

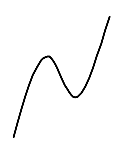
$x^2 = 9 \; |\sqrt{\;}$ $x_1 = 3$; $x_2 = -3$

$x^2 = 1 \; |\sqrt{\;}$ $x_3 = 1$; $x_4 = -1$

$S_{x1}(3|0)$ $S_{x2}(-3|0)$ $S_{x3}(1|0)$ $S_{x4}(-1|0)$



e) $f(x) = \frac{1}{3}x^3 + \frac{5}{3}x^2 + x - 3$ 1. $D = R$ 2. $x \to -\infty; f(x) \to -\infty$; $x \to +\infty; f(x) \to +\infty$



3. KS

4. $S_y(0|-3)$ $f(x) = 0$ $0 = \frac{1}{3}x^3 + \frac{5}{3}x^2 + x - 3 \; \Big| : \frac{1}{3}$

$0 = x^3 + 5x^2 + 3x - 9$ Polynomdivision mit $x_1 = 1$

$$(x^3 + 5x^2 + 3x - 9) : (x - 1) = x^2 + 6x + 9$$

$$\underline{-(x^3 - x^2)}$$

$$6x^2 + 3x$$

$$\underline{-(6x^2 - 6x)}$$

$$9x - 9$$

$$\underline{-(9x - 9)}$$

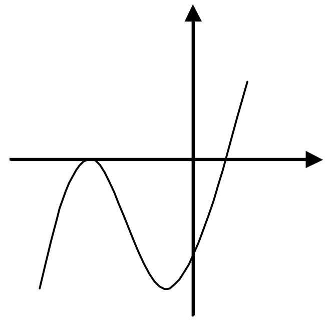
$$0$$

$$x^2 + 6x + 9 = 0$$

$$x_{2/3} = -3 \pm \sqrt{9 - 9}$$

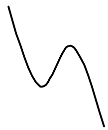
$$x_{2/3} = -3$$

5.



$S_{x1}(1|0)$ $S_{x2/3}(-3|0)$

f) $f(x) = -0{,}5x^3 + 4$ 1. $D = R$ 2. $x \to -\infty; f(x) \to +\infty$; $x \to +\infty; f(x) \to -\infty$



3. KS

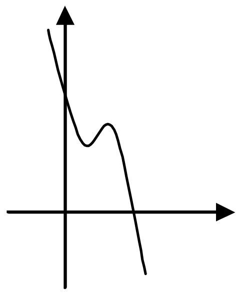
4. $S_y(0|4)$ $f(x) = 0$ $0 = -0{,}5x^3 + 4 \,|\, : (-0{,}5)$

$$0 = x^3 - 8 \,|\, + 8$$

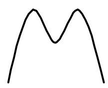
$$x^3 = 8 \,|\, \sqrt[3]{\;}$$

$$x_1 = 2$$

$S_{x1}(2|0)$



g) $f(x) = -x^4 + 3x^2 + 4$ 1. $D = R$ 2. $x \to -\infty; f(x) \to -\infty$; $x \to +\infty; f(x) \to -\infty$



3. AS

4. $S_y(0|4)$ $f(x) = 0$ $0 = -x^4 + 3x^2 + 4 \,|\, : (-1)$

$0 = x^4 - 3x^2 - 4$ Substitution

$$x^2 = z$$

$$0 = z^2 - 3z - 4$$

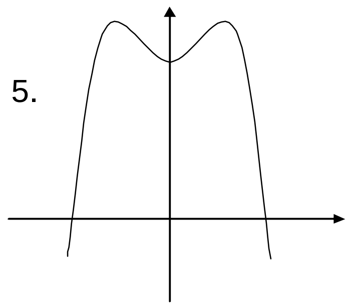
$$z_{1/2} = +1{,}5 \pm \sqrt{2{,}25 + 4}$$

$$z_1 = 4 \quad ; \quad z_2 = -1$$

$x^2 = 4 \,|\sqrt{\;}$ $x_1 = 2$; $x_2 = -2$

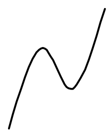
$x^2 = -1 \,|\sqrt{\;}$ nicht lösbar

5.



$S_{x1}(2|0)$ $S_{x2}(-2|0)$

h) $f(x) = 2x^3 - 2x^2 - 12x$ 1. $D = R$ 2. $x \to -\infty; f(x) \to -\infty$; $x \to +\infty; f(x) \to +\infty$



3. KS

4. $S_y(0|0)$ $f(x) = 0$ $0 = 2x^3 - 2x^2 - 12x \,|\, : 2$

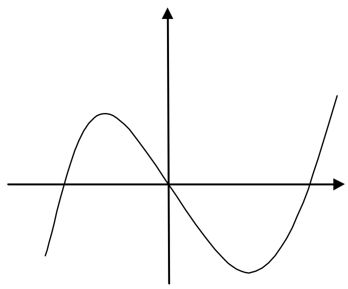
$$0 = x^3 - x^2 - 6x$$

$$0 = x(x^2 - x - 6)$$

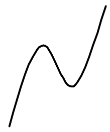
$$x_1 = 0 \quad ; \quad x^2 - x - 6 = 0$$

p-q-Formel $x_2 = 3$; $x_3 = -2$

$S_{x1}(0|0)$ $S_{x2}(3|0)$ $S_{x3}(-2|0)$



i) $f(x)=\frac{1}{5}x^3-3{,}8x+6$ 1. $D=R$ 2. $x\to-\infty; f(x)\to-\infty$; $x\to+\infty; f(x)\to+\infty$



3. KS

4. $S_y(0|6)$ $f(x)=0$ $0=\frac{1}{5}x^3-3{,}8x+6 \;|:\frac{1}{5}$

$0=x^3-19x+30$ Polynomdivision mit $x_1=2$

$(x^3+0x^2-19x+30):(x-2)=x^2+2x-15$

$-(x^3-2x^2)$

$2x^2-19x$

$-(2x^2-4x)$

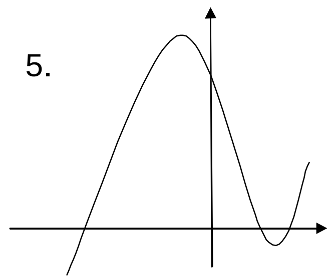
$-15x+30$

$-(-15x+30)$

0

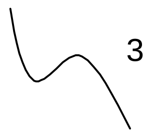
p-q-Formel $x_2=3$; $x_3=-5$

5.



$S_{x1}(2|0)$ $S_{x2}(3|0)$ $S_{x3}(-5|0)$

j) $f(x)=-0{,}5x^5+3x^4-4{,}5x^3$ 1. $D=R$ 2. $x\to-\infty; f(x)\to+\infty$; $x\to+\infty; f(x)\to-\infty$



3. KS

4. $S_y(0|0)$ $f(x)=0$ $0=-0{,}5x^5+3x^4-4{,}5x^3 \;|:(-0{,}5)$

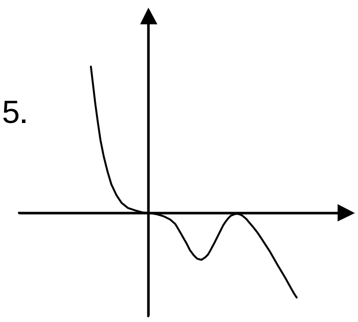
$0=x^5-6x^4+9x^3$

$0=x^3(x^2-6x+9)$

$x_{1/2/3}=0$; $x^2-6x+9=0$

p-q-Formel $x_{4/5}=3$

5.



$S_{x1/2/3}(0|0)$ $S_{x4/5}(3|0)$

<u>2. Aufgabe</u>

a) $f_1(x)=f_2(x)$

$x^3-8x^2+16x=0{,}4x^3-2{,}6x^2+4x \;|-0{,}4x^3+2{,}6x^2-4x$

$0{,}6x^3-5{,}4x^2+12x=0 \;|:0{,}6$

$x^3-9x^2+20x=0$ x ausklammern und p-q-Formel

$x_1=0$; $x_2=4$; $x_3=5$

Bei Schnittpunkten wird der zugehörige y-Wert in einer der beiden Ausgangsgleichungen berechnet.

$f_2(0)=0$ $f_2(4)=0$ $f_1(5)=5$

$S_1(0|0)$ $S_2(4|0)$ $S_3(5|5)$

b) $f_1(x)=f_2(x)$

$2x^3-3x=3x^2-2 \;|-3x^2+2$

$2x^3-3x^2-3x+2=0$ Polynomdivision mit $x_1=-1$

p-q-Formel $x_2=2$; $x_3=0{,}5$

$f_2(-1)=1$ $f_2(2)=10$ $f_1(0{,}5)=-1{,}25$

$S_1(-1|1)$ $S_2(2|10)$ $S_3(0{,}5|-1{,}25)$

c) $f_1(x) = f_2(x)$

$2x^4 - 6x = -2x^2 - 6x + 4 \; | + 2x^2 + 6x - 4$

$2x^4 + 2x^2 - 4 = 0 \; | : 2$

$x^4 + x^2 - 2 = 0$ Substitution mit $x^2 = z$, dann p-q , dann $z = x^2$

$x^2 = 1 \; |\sqrt{\;}$ $x_1 = 1$; $x_2 = -1$

$x^2 = -2 \; |\sqrt{\;}$ nicht lösbar

$f_1(1) = -4$ $f_1(-1) = 8$

$S_1(1|-4)$ $S_2(-1|8)$

d) $f_1(x) = f_2(x)$

$2{,}1x^3 - 4x = 1{,}1x^3 + 2{,}8x^2 - 2{,}6x \; | - 1{,}1x^3 - 2{,}8x^2 + 2{,}6x$

$x^3 - 2{,}8x^2 - 1{,}4x = 0$ x ausklammern und p-q-Formel

$x_1 = 0$; $x_2 = 3{,}2$; $x_3 = -0{,}4$

$f_1(0) = 0$ $f_1(3{,}2) = 56{,}0$ $f_1(-0{,}4) = 1{,}5$ vgl. $f_2(3{,}2) = 56{,}4$ und $f_2(-0{,}4) = 1{,}4$

$S_1(0|0)$ $S_2(3{,}2|56)$ $S_3(-0{,}4|1{,}5)$

Da die x-Werte gerundet wurden, entsteht eine Abweichung bei den y-Werten in beiden Funktionen. Daher ist es absolut notwendig anzugeben, in welche der Funktionen eingesetzt wurde. $f_1(x)$ oder $f_2(x)$