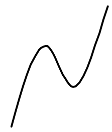
# Lösungen B 14

<u>1. Aufgabe</u>

**a)** $f(x)=\frac{1}{4}x^3+x^2-\frac{1}{4}x-1$ 1. $D=\mathbb{R}$ 2. $x\to-\infty;\ f(x)\to-\infty$ ; $x\to+\infty;\ f(x)\to+\infty$



3. KS

4. $S_y(0|-1)$ $f(x)=0$ $0=\frac{1}{4}x^3+x^2-\frac{1}{4}x-1 \quad \Big| : \frac{1}{4}$

$0=x^3+4x^2-x-4$ Polynomdivision mit $x_1=1$

$$\begin{aligned}
&(x^3+4x^2-x-4):(x-1)=x^2+5x+4\\
&\underline{-(x^3-1x^2)}\\
&\quad 5x^2-x\\
&\quad \underline{-(5x^2-5x)}\\
&\qquad 4x-4\\
&\qquad \underline{-(4x-4)}\\
&\qquad\quad 0
\end{aligned}$$

$x^2+5x+4=0$

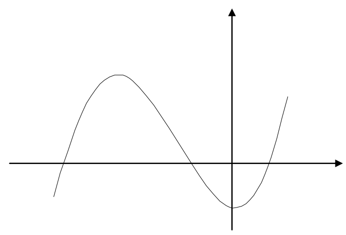
$x_{2/3}=-2{,}5\pm\sqrt{2{,}5^2-4}$

$x_2=-1$

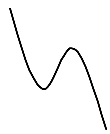
$x_3=-4$

$S_{x1}(1|0)$ $S_{x2}(-1|0)$ $S_{x3}(-4|0)$

5.



**b)** $f(x)=-0{,}5x^3+4{,}5x$ 1. $D=\mathbb{R}$ 2. $x\to-\infty;\ f(x)\to+\infty$ ; $x\to+\infty;\ f(x)\to-\infty$



3. PS

4. $S_y(0|0)$ $f(x)=0$ $0=-0{,}5x^3+4{,}5x \quad \big| :(-0{,}5)$

$0=x^3-9x$ x ausklammern

$0=x(x^2-9)$

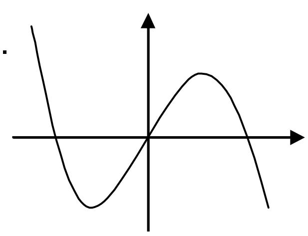
$x_1=0$ ; $x^2-9=0 \quad \big| +9$

$x^2=9 \quad \big| \sqrt{\ }$

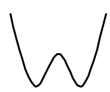
$x_2=3$ ; $x_3=-3$

$S_{x1}(0|0)$ $S_{x2}(3|0)$ $S_{x3}(-3|0)$

5.



**c)** $f(x)=\frac{1}{4}x^4-\frac{7}{4}x^3+3x^2+x-4$ 1. $D=\mathbb{R}$ 2. $x\to-\infty;\ f(x)\to+\infty$ ; $x\to+\infty;\ f(x)\to+\infty$



3. KS

4. $S_y(0|-4)$ $f(x)=0$ $0=\frac{1}{4}x^4-\frac{7}{4}x^3+3x^2+x-4 \Big| : \frac{1}{4}$

$0=x^4-7x^3+12x^2+4x-16$ Polynomdivision mit $x_1=-1$

$$(x^4 - 7x^3 + 12x^2 + 4x - 16) : (x+1) = x^3 - 8x^2 + 20x - 16$$
$$\underline{-(x^4 + x^3)}$$
$$-8x^3 + 12x^2$$
$$\underline{-(-8x^3 - 8x^2)}$$
$$20x^2 + 4x$$
$$\underline{-(20x^2 + 20x)}$$
$$-16x - 16$$
$$\underline{-(-16x - 16)}$$
$$0$$

$x^3 - 8x^2 + 20x - 16 = 0$ Polynomdivision mit $x_2 = 2$

$$(x^3 - 8x^2 + 20x - 16) : (x-2) = x^2 - 6x + 8$$
$$\underline{-(x^3 - 2x^2)}$$
$$-6x^2 + 20x$$
$$\underline{-(-6x^2 + 12x)}$$
$$8x - 16$$
$$\underline{-(8x - 16)}$$
$$0$$

$x^2 - 6x + 8 = 0$

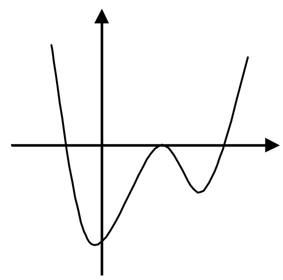
$x_{3/4} = +3 \pm \sqrt{9-8}$

$x_3 = 4$

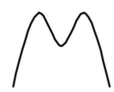
$x_4 = 2$ doppelte Lösung

$S_{x1}(-1|0)$ $S_{x2/4}(2|0)$ $S_{x3}(4|0)$

5.



**d)** $f(x) = -0{,}1x^4 + x^2 - 0{,}9$ 1. $\mathrm{D} = \mathrm{R}$ 2. $x \to -\infty;\ f(x) \to -\infty$ ; $x \to +\infty;\ f(x) \to -\infty$



3. AS

4. $S_y(0|-0{,}9)$ $f(x) = 0$ $0 = -0{,}1x^4 + x^2 - 0{,}9 \ \big|: (-0{,}1)$

$0 = x^4 - 10x^2 + 9$ biquadratische Gleichung, Substitution

$x^2 = z$

$0 = z^2 - 10z + 9$ p-q-Formel

$z_{1/2} = +5 \pm \sqrt{5^2 - 9}$

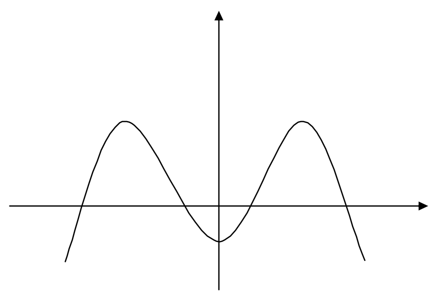
$z_1 = 9$

$z_2 = 1$

$z = x^2$ Resubstitution

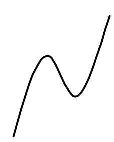
$x^2 = 9 \ \big|\sqrt{\ }$ $x_1 = 3$ ; $x_2 = -3$

$x^2 = 1 \ \big|\sqrt{\ }$ $x_3 = 1$ ; $x_4 = -1$



$S_{x1}(3|0)$ $S_{x2}(-3|0)$ $S_{x3}(1|0)$ $S_{x4}(-1|0)$

**e)** $f(x) = \frac{1}{3}x^3 + \frac{5}{3}x^2 + x - 3$ 1. $\mathrm{D} = \mathrm{R}$ 2. $x \to -\infty;\ f(x) \to -\infty$ ; $x \to +\infty;\ f(x) \to +\infty$



3. KS

4. $S_y(0|-3)$ $f(x) = 0$ $0 = \frac{1}{3}x^3 + \frac{5}{3}x^2 + x - 3 \ \Big|: \frac{1}{3}$

$0 = x^3 + 5x^2 + 3x - 9$ Polynomdivision mit $x_1 = 1$

$(x^3 + 5x^2 + 3x - 9) : (x-1) = x^2 + 6x + 9$

$\underline{-(x^3 - x^2)}$

$6x^2 + 3x$

$\underline{-(6x^2 - 6x)}$

$9x - 9$

$\underline{-(9x-9)}$

$0$

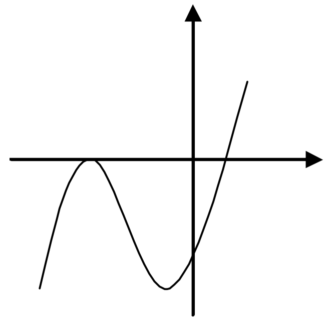
$x^2 + 6x + 9 = 0$

$x_{2/3} = -3 \pm \sqrt{9-9}$

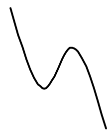
$x_{2/3} = -3$

$S_{x1}(1|0) \qquad S_{x2/3}(-3|0)$

5.



**f)** $f(x) = -0{,}5x^3 + 4$ 1. $D = R$ 2. $x \to -\infty;\ f(x) \to +\infty$; $x \to +\infty;\ f(x) \to -\infty$



3. KS

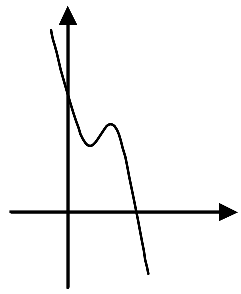
4. $S_y(0|4)$ $f(x) = 0$ $0 = -0{,}5x^3 + 4 \,|: (-0{,}5)$

$0 = x^3 - 8 \,| + 8$

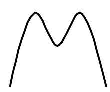
$x^3 = 8 \,|\sqrt[3]{\ }$

$x_1 = 2$

$S_{x1}(2|0)$



**g)** $f(x) = -x^4 + 3x^2 + 4$ 1. $D = R$ 2. $x \to -\infty;\ f(x) \to -\infty$; $x \to +\infty;\ f(x) \to -\infty$



3. AS

4. $S_y(0|4)$ $f(x) = 0$ $0 = -x^4 + 3x^2 + 4 \,|: (-1)$

$0 = x^4 - 3x^2 - 4$ Substitution

$x^2 = z$

$0 = z^2 - 3z - 4$

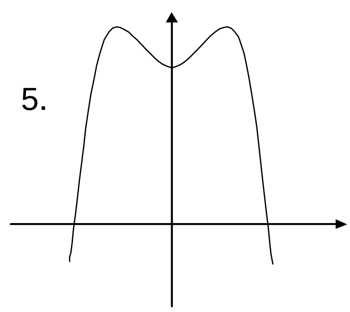
$z_{1/2} = +1{,}5 \pm \sqrt{2{,}25 + 4}$

$z_1 = 4 \quad ; \quad z_2 = -1$

$x^2 = 4 \,|\sqrt{\ }$ $\qquad x_1 = 2 \quad ; \quad x_2 = -2$

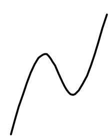
$x^2 = -1 \,|\sqrt{\ }$ nicht lösbar

5.



$S_{x1}(2|0) \qquad S_{x2}(-2|0)$

**h)** $f(x) = 2x^3 - 2x^2 - 12x$ 1. $D = R$ 2. $x \to -\infty;\ f(x) \to -\infty$; $x \to +\infty;\ f(x) \to +\infty$



3. KS

4. $S_y(0|0)$ $f(x) = 0$ $0 = 2x^3 - 2x^2 - 12x \,|: 2$

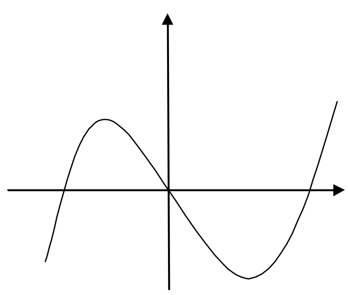
$0 = x^3 - x^2 - 6x$

$0 = x(x^2 - x - 6)$

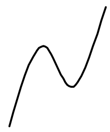
$x_1 = 0 \quad ; \quad x^2 - x - 6 = 0$

p-q-Formel $\qquad x_2 = 3 \quad ; \quad x_3 = -2$

$S_{x1}(0|0) \qquad S_{x2}(3|0) \qquad S_{x3}(-2|0)$



**i)** $f(x)=\frac{1}{5}x^3-3{,}8x+6$ 1. $D=R$ 2. $x\to-\infty;\ f(x)\to-\infty$ ; $x\to+\infty;\ f(x)\to+\infty$



3. KS

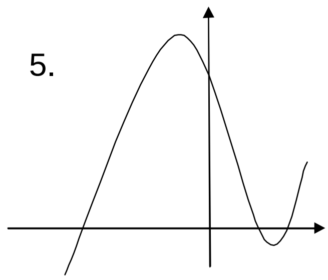
4. $S_y(0|6)$ $f(x)=0$ $0=\frac{1}{5}x^3-3{,}8x+6 \ \big|:\frac{1}{5}$

$0=x^3-19x+30$ Polynomdivision mit $x_1=2$

$$(x^3+0x^2-19x+30):(x-2)=x^2+2x-15$$
$$\underline{-(x^3-2x^2)}$$
$$2x^2-19x$$
$$\underline{-(2x^2-4x)}$$
$$-15x+30$$
$$\underline{-(-15x+30)}$$
$$0$$

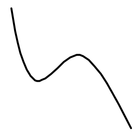
p-q-Formel $x_2=3$ ; $x_3=-5$

5.



$S_{x1}(2|0)$ $S_{x2}(3|0)$ $S_{x3}(-5|0)$

**j)** $f(x)=-0{,}5x^5+3x^4-4{,}5x^3$ 1. $D=R$ 2. $x\to-\infty;\ f(x)\to+\infty$ ; $x\to+\infty;\ f(x)\to-\infty$



3. KS

4. $S_y(0|0)$ $f(x)=0$ $0=-0{,}5x^5+3x^4-4{,}5x^3 \big|:(-0{,}5)$

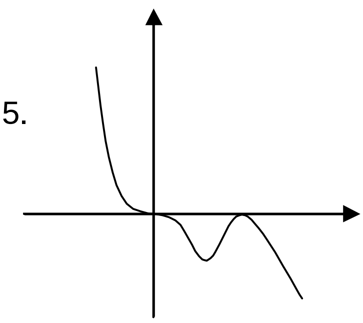
$0=x^5-6x^4+9x^3$

$0=x^3(x^2-6x+9)$

$x_{1/2/3}=0$ ; $x^2-6x+9=0$

p-q-Formel $x_{4/5}=3$

5.



$S_{x1/2/3}(0|0)$ $S_{x4/5}(3|0)$

2. Aufgabe

a) $f_1(x)=f_2(x)$

$x^3-8x^2+16x=0{,}4x^3-2{,}6x^2+4x \ \big|-0{,}4x^3+2{,}6x^2-4x$

$0{,}6x^3-5{,}4x^2+12x=0 \ \big|:0{,}6$

$x^3-9x^2+20x=0$ x ausklammern und p-q-Formel

$x_1=0$ ; $x_2=4$ ; $x_3=5$

Bei Schnittpunkten wird der zugehörige y-Wert in einer der beiden Ausgangsgleichungen berechnet.

$f_2(0)=0$ $f_2(4)=0$ $f_1(5)=5$

$S_1(0|0)$ $S_2(4|0)$ $S_3(5|5)$

b) $f_1(x)=f_2(x)$

$2x^3-3x=3x^2-2 \ \big|-3x^2+2$

$2x^3-3x^2-3x+2=0$ Polynomdivision mit $x_1=-1$

p-q-Formel $x_2=2$ ; $x_3=0{,}5$

$f_2(-1)=1$ $f_2(2)=10$ $f_1(0{,}5)=-1{,}25$

$S_1(-1|1)$ $S_2(2|10)$ $S_3(0{,}5|-1{,}25)$

c) $f_1(x) = f_2(x)$

$2x^4 - 6x = -2x^2 - 6x + 4 \quad | + 2x^2 + 6x - 4$

$2x^4 + 2x^2 - 4 = 0 \quad | : 2$

$x^4 + x^2 - 2 = 0$ Substitution mit $x^2 = z$ , dann p-q , dann $z = x^2$

$x^2 = 1 \quad | \sqrt{\ }$ $x_1 = 1$ ; $x_2 = -1$

$x^2 = -2 \quad | \sqrt{\ }$ nicht lösbar

$f_1(1) = -4$ $f_1(-1) = 8$

$S_1(1|-4)$ $S_2(-1|8)$

d) $f_1(x) = f_2(x)$

$2{,}1x^3 - 4x = 1{,}1x^3 + 2{,}8x^2 - 2{,}6x \quad | -1{,}1x^3 - 2{,}8x^2 + 2{,}6x$

$x^3 - 2{,}8x^2 - 1{,}4x = 0$ x ausklammern und p-q-Formel

$x_1 = 0$ ; $x_2 = 3{,}2$ ; $x_3 = -0{,}4$

$f_1(0) = 0$ $f_1(3{,}2) = 56{,}0$ $f_1(-0{,}4) = 1{,}5$ vgl. $f_2(3{,}2) = 56{,}4$ und $f_2(-0{,}4) = 1{,}4$

$S_1(0|0)$ $S_2(3{,}2|56)$ $S_3(-0{,}4|1{,}5)$

Da die x-Werte gerundet wurden, entsteht eine Abweichung bei den y-Werten in beiden Funktionen. Daher ist es absolut notwendig anzugeben, in welche der Funktionen eingesetzt wurde. $f_1(x)$ *oder* $f_2(x)$