

## Lösungen A 13

a)  $x^3 - 4x^2 + x + 6 = 0$  Polynomdivision mit  $x_1 = -1$

$$(x^3 - 4x^2 + x + 6) : (x + 1) = x^2 - 5x + 6$$

$$\underline{-(x^3 + 1x^2)}$$

$$-5x^2 + x$$

$$\underline{-(-5x^2 - 5x)}$$

$$6x + 6$$

$$\underline{-(6x + 6)}$$

$$0$$

$$x^2 - 5x + 6 = 0$$

p-q-Formel

$$x_{2/3} = +2,5 \pm \sqrt{2,5^2 - 6}$$

$$x_2 = 3$$

$$x_3 = 2$$

b)  $x^4 - 29x^2 + 100 = 0$  biquadratische Gleichung, Substitution mit

$$x^2 = z$$

$$z^2 - 29z + 100 = 0 \quad \text{p-q-Formel}$$

$$z_{1/2} = +14,5 \pm \sqrt{14,5^2 - 100}$$

$$z_1 = 25$$

$$z_2 = 4$$

Resubstitution mit

$$z = x^2$$

$$x^2 = 25 \quad | \sqrt{\quad} \quad x_1 = 5 \quad \vee \quad x_2 = -5$$

$$x^2 = 4 \quad | \sqrt{\quad} \quad x_3 = 2 \quad \vee \quad x_4 = -2$$

c)  $x^3 - 5x^2 + 3x = 0$  Ausklammern von  $x$

$$x \cdot (x^2 - 5x + 3) = 0$$

$$x_1 = 0 \quad \vee \quad x^2 - 5x + 3 = 0 \quad \text{p-q-Formel} \quad (\text{Anmerkung: Das Zeichen } \vee \text{ heißt oder})$$

$$x_{2/3} = +2,5 \pm \sqrt{2,5^2 - 3}$$

$$x_2 = 4,3$$

$$x_3 = 0,7$$

d)  $x^3 + 6x^2 = 0$  Ausklammern von  $x^2$

$$x^2 \cdot (x + 6) = 0$$

$$x^2 = 0 \quad | \sqrt{\quad} \quad \vee \quad x + 6 = 0 \quad | -6$$

$$x_{1/2} = 0 \quad \vee \quad x_3 = -6$$

e)  $x^4 - 81 = 0 \quad | +81$

$$x^4 = 81 \quad | \sqrt{\quad}$$

$$x_1 = 3 \quad \vee \quad x_2 = -3$$

f)  $0,4x^3 + 1,2x^2 - 5,2x - 6 = 0$  Polynomdivision mit  $x_1 = -1$

$(0,4x^3 + 1,2x^2 - 5,2x - 6) : (x+1) = 0,4x^2 + 0,8x - 6$

$-(0,4x^3 + 0,4x^2)$

$0,8x^2 - 5,2x$

$-(0,8x^2 + 0,8x)$

$-6x - 6$

$-( -6x - 6)$   
 $0$

$0,4x^2 + 0,8x - 6 = 0 \quad | :0,4$

$x^2 + 2x - 15 = 0$

$x_{2/3} = -1 \pm \sqrt{1^2 + 15}$

$x_2 = 3$

$x_3 = -5$

p-q-Formel

g)  $-\frac{1}{4}x^3 - x^2 - \frac{1}{4}x + 1,5 = 0 \quad | :(-\frac{1}{4})$

$x^3 + 4x^2 + x - 6 = 0$

$x_1 = 1$

$(x^3 + 4x^2 + x - 6) : (x-1) = x^2 + 5x + 6$

$-(x^3 - x^2)$

$5x^2 + x$

$-(5x^2 - 5x)$

$6x - 6$

$-(6x - 6)$   
 $0$

$x^2 + 5x + 6 = 0$

$x_{2/3} = -2,5 \pm \sqrt{2,5^2 - 6}$

$x_2 = -2$

$x_3 = -3$

h)  $-0,5x^4 + 5x^2 - 4,5 = 0 \quad | :(-0,5)$

$x^4 - 10x^2 + 9 = 0$

Substitution

$x^2 = z$

$z^2 - 10z + 9 = 0$

$z_{1/2} = 5 \pm \sqrt{5^2 - 9}$

$z_1 = 9$

$z_2 = 1$

$z = x^2$

$x^2 = 9 \quad | \sqrt{\quad} \quad x_1 = 3 \quad \vee \quad x_2 = -3$

$x^2 = 1 \quad | \sqrt{\quad} \quad x_3 = 1 \quad \vee \quad x_4 = -1$

i)  $\frac{1}{5}x^3 - x - 20 = 0 \quad | : \frac{1}{5}$

$x^3 - 5x - 100 = 0$

$x_1 = 5$

$(x^3 + 0x^2 - 5x - 100) : (x-5) = x^2 + 5x + 20$

$-(x^3 - 5x^2)$

$5x^2 - 5x$

$-(5x^2 - 25x)$

$20x - 100$

$-(20x - 100)$   
 $0$

$x^2 + 5x + 20 = 0$

$x_{2/3} = -2,5 \pm \sqrt{2,5^2 - 20}$

nicht lösbar

$$\begin{aligned}
 \text{j)} \quad & -x^2 + 81 = 0 \\
 & x^2 = 81 \quad | \sqrt{\phantom{x}} \\
 & x_1 = 9 \\
 & x_2 = -9
 \end{aligned}$$

$$\begin{aligned}
 \text{k)} \quad & 0,5x^4 - 8x^2 = 0 \\
 & x^4 - 16x^2 = 0 \\
 & x^2(x^2 - 16) = 0 \\
 & x_{1/2} = 0 \quad \vee \quad x^2 - 16 = 0 \\
 & \qquad \qquad \qquad x_3 = 4 \\
 & \qquad \qquad \qquad x_4 = -4
 \end{aligned}$$

$$\begin{aligned}
 \text{l)} \quad & \frac{5}{6}x + 15 = 0 \quad | -15 \\
 & \frac{5}{6}x = -15 \quad | : \frac{5}{6} \\
 & x = -18
 \end{aligned}$$

$$\begin{aligned}
 \text{m)} \quad & 3x^3 - 8,5x^2 - 4,5 = 0 \quad | \cdot 2 & \text{Teilersuche schwierig, daher mal 2} \\
 & 6x^3 - 17x^2 - 9 = 0 & x_1 = 3 \\
 & (6x^3 - 17x^2 + 0x - 9) : (x - 3) = 6x^2 + x + 3 \\
 & \begin{array}{r}
 -(6x^3 - 18x^2) \\
 \hline
 x^2 + 0x \\
 -(x^2 - 3x) \\
 \hline
 3x - 9 \\
 -(3x - 9) \\
 \hline
 0
 \end{array} & \begin{aligned}
 6x^2 + x + 3 = 0 \quad | : 6 \\
 x^2 + \frac{1}{6}x + 0,5 = 0 \\
 x_{2/3} = -\frac{1}{12} \pm \sqrt{\left(\frac{1}{12}\right)^2 - 0,5}
 \end{aligned}
 \end{aligned}$$

*nicht lösbar*

$$\begin{aligned}
 \text{n)} \quad & \frac{3}{4}x^3 - \frac{1}{4}x^2 + \frac{3}{2}x - 2 = 0 & x_1 = 1 \\
 & \left(\frac{3}{4}x^3 - \frac{1}{4}x^2 + \frac{3}{2}x - 2\right) : (x - 1) = \frac{3}{4}x^2 + \frac{1}{2}x + 2 \\
 & \begin{array}{r}
 -\left(\frac{3}{4}x^3 - \frac{3}{4}x^2\right) \\
 \hline
 \frac{1}{2}x^2 + \frac{3}{2}x \\
 -\left(\frac{1}{2}x^2 - \frac{1}{2}x\right) \\
 \hline
 2x - 2 \\
 -(2x - 2) \\
 \hline
 0
 \end{array} & \begin{aligned}
 \frac{3}{4}x^2 + \frac{1}{2}x + 2 = 0 \quad | : \frac{3}{4} \\
 x^2 + \frac{2}{3}x + \frac{8}{3} = 0 \\
 x_{2/3} = -\frac{1}{3} \pm \sqrt{\left(\frac{1}{3}\right)^2 - \frac{8}{3}}
 \end{aligned}
 \end{aligned}$$

*nicht lösbar*

$$\begin{aligned}
 & 0 \\
 \text{o)} \quad & 4x^4 - 6x^3 = 0 \\
 & x^3(4x - 6) = 0 \\
 & x^3 = 0 \quad \sqrt[3]{\phantom{x}} \quad \vee \quad 4x - 6 = 0 \\
 & x_{1/2/3} = 0 \qquad \qquad x_4 = 1,5
 \end{aligned}$$

$$\begin{aligned}
 \text{p)} \quad & \frac{1}{2}x^3 - \frac{1}{4}x^2 - 7x = 0 \quad | : \frac{1}{2} \\
 & x^3 - 0,5x^2 - 14x = 0 \\
 & x(x^2 - 0,5x - 14) = 0 \\
 & x_1 = 0 \quad \vee \quad x^2 - 0,5x - 14 = 0 \\
 & \qquad \qquad \qquad x_{2/3} = 0,25 \pm \sqrt{0,25^2 + 14} \\
 & \qquad \qquad \qquad x_2 = 4 \\
 & \qquad \qquad \qquad x_3 = -3,5
 \end{aligned}$$

$$\begin{aligned}
 \text{q)} \quad & -3x^4 + 21x^2 - 36 = 0 \quad | : (-3) \\
 & x^4 - 7x^2 + 12 = 0 \\
 & \qquad \qquad \qquad x^2 = z \\
 & z^2 - 7z + 12 = 0 \\
 & z_{1/2} = 3,5 \pm \sqrt{3,5^2 - 12} \\
 & z_1 = 4 \\
 & z_2 = 3 \\
 & z = x^2 \\
 & x^2 = 4 \quad \sqrt{\phantom{x}} \qquad \qquad x_1 = 2 \qquad \vee \qquad x_2 = -2 \\
 & x^2 = 3 \quad \sqrt{\phantom{x}} \qquad \qquad x_3 = 1,7 \qquad \qquad x_4 = -1,7
 \end{aligned}$$